TÉCNICO LISBOA

Optimization of Maintenance and Availability in a Train Operating Company

Fertagus Case Study

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Abstract

Railway systems in Europe are facing, nowadays, a wide range of measures that will hopefully bring till 2050 a new competitiveness and a key strategic importance to the sector, revolutionizing its importance in transportation. Herewith a lot of research will be induced, to apply new approaches. An optimized rolling-stock planning, associated with a sustainable reduction of costs, improvement of service reliability and adapted to the current customer demand and maintenance requirements is of high potential, and therefore, one way to respect budget goals, while new costs in research and development are added. A mixed-integer linear programming decision model, which considers the preventive maintenance actions that must be performed for each week of the year, is presented and gives a weekly rolling-stock schedule. It minimizes the operational costs, while adding maintenance actions to the roster, and is validated by an illustrative example of a small sized problem. Afterwards several real instances applied to Portuguese Railway operating company Fertagus are solved to optimality. Only a 3-day schedule was possible to run due to computational capacity limitations. Moreover, sensitivity analysis on the weights of the different components of the objective function demonstrated that the optimal solution found is not sensitive to significant variations of the weights. An intermediate model that links the 1-day operational planning model proposed and the annual tactical plan, able to reduce the size of the problem, and a comprehensive crew scheduling that considers the different skill of maintenance technicians and their experience, are proposed for further research.

Key Words: Railway Management, Rolling-Stock Planning, Maintenance Scheduling, Robustness, Optimization, Mixed-Integer Linear Programming

Resumo

O sistema ferroviário europeu enfrenta atualmente um conjunto de medidas, que até 2050, se espera levar a um aumento da competitividade e relevância estratégica do sector, fundamentais para revolucionar a sua importância no transporte. Nesse sentido, está intrínseco um aumento da investigação associada à inovação e desenvolvimento de novas soluções, bem como o investimento associado. Uma forma de compensar os custos deste investimento, é através de um planeamento otimizado, capaz de promover uma redução sustentável de custos, bem como uma melhoria da fiabilidade do serviço, e adaptado às necessidades do cliente e serviços de manutenção. Nesse sentido, é apresentado um modelo de decisão de programação linear inteira mista, que baseado nas ações de manutenção preventiva a serem executadas em cada semana do ano, fornece um cronograma semanal das tarefas a desempenhar pela frota. O modelo minimiza custos operacionais e adiciona simultaneamente ações de manutenção ao cronograma. É validado por um pequeno exemplo ilustrativo e posteriormente aplicado à empresa ferroviária portuguesa, Fertagus. Não foi possível correr o modelo para mais de 3 dias, devido a limitações de capacidade computacional. Análises de sensibilidade dos pesos dos diferentes termos da função objetivo demonstraram que a solução ótima encontrada não é sensível a variações consideráveis destes pesos. São propostos para investigação futura, um modelo intermédio que ligue o modelo de planeamento operacional diário e o modelo de planeamento tático anual, reduzindo assim a dimensão do problema, e um cronograma de equipas exaustivo, tendo em conta as diferentes capacidades e experiência dos técnicos de manutenção.

Palavras-chave: Planeamento Ferroviário, Planeamento de Unidades Ferroviárias, Planeamento da Manutenção, Robustez, Otimização, Programação Linear Inteira Mista

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List of Acronyms

SNFC – *Société nationale des chemins de fer français*

CP – *Comboios de Portugal*

BVT – Beginning Virtual Tasks

EVT – Ending Virtual Tasks

PMC – *Parque de Material de Coina*

MILP – Integer Linear Programming

ILP – Integer Linear Programming

EU – European Comission

1 Introduction

In this initial chapter, the research topic is briefly presented, through a contextualization of the railway systems in Europe and an exposition of the railway operations management, and then, an introduction of the present case study is provided. Afterwards, the research problem is identified, and the proposed approach and methodology are stated within the document's structure.

1.1 Context

1.1.1 Railway Mobility in Europe

Transportation systems in Europe are facing, nowadays, a wide range of profound structural reforms, related to a crescent strategic importance of this sector. Transportation is one of the cornerstones of the European economy. *"Transport is fundamental to our economy and society. Mobility is vital for growth and job creation. (…) Effective transport systems are key to European companies' ability to compete in the world economy. (…) The quality of transport services has a major impact on people's quality of life."* (EU, 2011).

Moreover, there is a need for a more competitive and connected Europe, capable to face a growing competition in fast developing world transport markets, which according to the European Commission Memo, *"Congestion, both on the roads and in the sky, is a major concern."* The *"Freight transport activity is projected to increase, with respect to 2005, by around 40% in 2030 and by little over 80% by 2050."* and *"Passenger traffic would grow slightly less than freight transport: 34% by 2030 and 51% by 2050"* (European Commission, 2011). The carrying capacity of the railways is extremely large and its capacity is elastic. By adding or reducing the number of train carriages an adjustment of the demand is possible. Therefore, if its use is reasonably high, it is a cheap mode of transport, as most of the expenses of railway systems are fixed costs, and thus every increase in the railway traffic is followed by a decrease in the average cost.

Moreover, unlike trains, most of the vehicles need oil derivate products to operate, so the path to follow to satisfy the future needs must be very well thought out. The dependency on mostly imported oil is a major concern. In one hand, for a sustainable economic perspective, since *"Oil will become scarcer in future decades, sourced increasingly from unstable parts of the world. Oil prices are projected to more than double between 2005 levels and 2050 (59 $/barrel in 2005). Current events show the extreme volatility of oil prices. Transport has become more energy-efficient but still depends on oil for 96% of its energy needs."* (European Commission, 2011). In the other hand, to address the tremendous environmental impact of these fossil fuel. *"There is the need to drastically reduce world greenhouse gas emissions, with the goal of limiting climate change to 2ºC. Overall, by 2050, the EU needs to reduce emissions by 80–95% below 1990 levels in order to reach this goal"* (European Commission, 2011). Thus, railway transportation can be considered a smart and efficient mean of transportation, and more strategies should be studied to transform it into a more popular mean of transportation.

However, some circumstances, still prevent railway transport from being a more widely used mean of transportation, for instance the ideal of a Single Transport Area is not yet possible, because the *"Infrastructure is unequally developed in the eastern and western parts of the EU. In the new Member States there are currently only around 4 800 km of motorways and no purpose-built high-speed rail lines; the conventional railway lines are often in poor condition"* (European Commission, 2011). This problem will have to be addressed if the railway industry wants to become more competitive in the future. The railway transport is economical, quicker and best suited for carrying heavy and bulky cargo over long distances, but freight transport is still mainly done through road and maritime way.

To answer these raised problems, the current European transport system has been facing major challenges and *"at the heart of the Transport 2050 roadmap is the need for a transformation in the rail sector so that it becomes more attractive and succeeds in carrying a very significantly increased share of the market for passenger and freight over middle distances (>300 km) by 2050. At the same time the aim is to triple the length of the current high-speed rail network by 2030."* (European Comission, 2011).

To bring off these transformations, a lot of research, such as the research and innovation programme Horizon 2020 aims to promote for innovation, will have to be conducted. Naturally, research challenge may have additional costs for railways companies, and another challenge is to allocate these costs in their budget, in order to become possible to apply new approaches. An optimized rolling-stock planning, associated with a sustainable reduction of costs and improvement of service reliability is an activity with high potential, and therefore, this can be one of the ways to respect budget goals, while new investments in research and development are added to the costs. The present work lines up into this EU vision, even if it is only applied to a train operating company in Lisbon, Portugal.

1.1.2 Railway Operations Management

To context the problem that will be addressed in this dissertation, the railway operation management is briefly summarized. Railway operations management can be divided in three major operations: i) timetabling, ii) rolling-stock circulation planning to cover timetable and iii) crew scheduling to operate the rolling-stock (Huisman *et al.*, 2005). These operations are usually carried out separately by railway operators, but their interdependency is crucial for a proper management of the all operation. Furthermore, in case the infrastructure manager and the railway operating company are not the same, a good cooperation between both is of equal importance. This dissertation is concerned with the rolling-stock circulation planning to cover a given timetable.

Regarding passenger transportation systems, punctuality and reliability are essential elements that influence its quality. However, disturbances such as vehicles or infrastructure failures may occur and affect the fulfilment of the initial planning. If possible, an adaptation that respects the services sold to the costumers should be carried out. Otherwise, the company will incur in an unfulfillment of the planned services. That can either entail direct cost to the company, or at least have impact on the corporate's image. A way to minimize this potential and unpredictable disturbances is of major interest to the transportation industry and is also an issue of the current work. More specifically, to avoid disturbances and delays spreading in a railway company. In this context, a robust rolling-stock plan can deal with operational disturbance possibilities and limit the service quality deterioration. It usually takes place from one year to six months before operations, depending also on its complexity and size of the company. Furthermore, a robust transportation planning does not guarantee a scenario without any delays. It requires a collective work of all the actors.

The rolling-stock planning problem should desirably not be separated from maintenance planning problem and rail inspection scheduling problem (Peng *et al*., 2013). In transportation companies, maintenance has a critical impact on both safety and availability. A careful maintenance planning is meant to be a trade-off between cost reduction and overly performed approach. If a vehicle lacks maintenance, the failure of its components will occur more frequently and will be more unpredictable. More specifically, there are two kinds of maintenance. The preventive maintenance, which aims to preserve the healthy condition of equipment and prevent failure, and the corrective maintenance which handles equipment failure and recovers it to operational conditions. Preventive maintenance is required at a given mileage or at a given time interval. A careful maintenance planning has naturally a positive impact on costs reduction, but also on the reliability of its components. Furthermore, a reliable component does not compromise the fulfilment of the operational task, because of its failure. To conclude, maintenance planning and rolling-stock planning are indeed interdependent and should be planned accordingly to.

1.2 Problem Definition and Methodology

As shown, the various measures proposed by the European Commission up to 2050 will bring a new competitiveness and a key strategic importance to the rail sector, revolutionizing its importance in transportation. Herewith a lot of research will be induced, which will have significant costs that railway companies will have to bear, to keep abreast of change. One way to face this huge investment is through a more careful and robust planning of the schedules, adapted to the current customer demand and maintenance requirements, reducing the overall costs of the operation.

The objective of this research work is to conceive a decision model capable of looking at the operation in a train operating company and perform a rolling-stock planning, for each week of the year. It must take into account the timetable of the company’s operation activities and a schedule relative to the preventive maintenance activities of each week of the year. The operative and maintenance costs are to be minimized and the availability of units to be maximized. Therefore, it is intended to apply the decision model to the case study of Fertagus train operating company, using a preventive maintenance technical planning from (Méchain, 2017), which outputs a weekly schedule, contemplating the preventive maintenance actions that have to be performed for that week and with the smallest maintenance cost possible.

To achieve these goals, the following steps were pursued:

- Literature review on transports operations management and scheduling;
- Development and validation of the decision model based on an optimization model described in the literature review;
- Data collection and model implementation in a mixed-integer linear programming model;
- Analysis of results and discussion;
- Take conclusions, recognize limitations and define future work;

Based on this multi-step approach, a model from the reviewed articles was selected (Tréfond *et al.*, 2017), as an inspiration for the model to be developed. Some adjustments were than required, and an innovative contribution was added to the model, in particular with the inclusion of the maintenance needs.

1.3 Document Structure

The present dissertation is structured in seven chapters:

1. Introduction – In this initial chapter, the research topic is briefly presented, through a contextualization of the railway systems in Europe and an exposition of the railway operations management, and then the present case study is also presented. Afterwards, the research problem is identified, and the approach and methodology are stated within the document's structure.

2. Related Work – State of the Art – In chapter 2, a review is presented on the work done on maintenance scheduling. The first section is related to different means of transport, apart from trains. The second section is specific to railway companies and starts with a wide overview on the research done so far. Useful knowledge for the current research topic is given, and in a third section, all the information is summarized, to provide an overview on what are the main research opportunities for this dissertation.

3. A Mixed-Integer Linear Programming Model – In chapter 3, the optimization model is described and explained in detail. The rolling-stock planning problem is fully defined and all the data, the objective function, the decision variables and the associated constraints are also presented and discussed in detail.

4. Application – Model Implementation and Validation - In chapter 4, an illustrative example is presented in detail to provide a better understanding of the concepts introduced in the previous chapter. Through this implementation it is intended to test and validate the model. In the first section, the implementation in FICO Xpress is described. In the second section, the illustrative example and its parameters are defined, and, in the third section, the results of the mathematical model are analysed.

5. Case Study - Fertagus – In chapter 5, the Fertagus railway operating company is briefly described and the case study specifications are presented. The parameters of the mathematical model are displayed, similarly to the previous chapter.

6. Results and Discussion – In chapter 6, the results for comparison with Fertagus' current plan are firstly presented and discussed. Then, several results and analysis can be found. Tests were carried out, to assess the impact of the different weights used in the objective function.

7. Conclusion – In this final chapter, the conclusions of the performed research can be found. Some limitations are discussed, and possible steps of future research are presented.

1.4 Objectives

The objective of this research work is to conceive a mathematical decision model capable of looking at the operation in a train operating company and perform an optimal rolling-stock planning, for each week of the year, reducing the overall costs of the operation. It must take into account the timetable of the company's operation activities and a schedule relative to the preventive maintenance activities of each week of the year. The operative and maintenance costs are to be minimized and the availability of units to be maximized. Therefore, it is intended to apply the decision model to the case study of Fertagus train operating company, using a preventive maintenance technical planning from (Méchain, 2017), which outputs a weekly schedule, contemplating the preventive maintenance actions that have to be performed on each train unit and for that week and with the smallest maintenance cost possible.
An underlying objective is to verify that the program is able to give an optimal feasible solution to the rolling-stock planning problem.

2 Related Work - State of the Art

In chapter 2, a review is presented on the work done on maintenance scheduling. The first section is related to different means of transport, apart from trains. The second section is specific to railway companies and starts with a wide overview on the research done so far. Useful knowledge for the current research topic is given, and in a third section, all the information is summarized, to provide an overview on what are the main research opportunities for this dissertation.

2.1 Maintenance Scheduling in Transportation Companies

Haghani and Shafahi (2002) focused on finding a way to respond to the problem of scheduling bus maintenance. The goal of the paper is to perform buses' maintenance during their idle time, to reduce the number of hours that the vehicle is out of the service, in other words, to reduce unavailability. Through an integer programming approach, a maintenance schedule for each bus is obtained, as well as the minimum number of maintenance lines that are used for each type of inspection.

Kozanidis and Skipis (2006) modeled an aircraft maintenance problem, which the objective was to maximize fleet's availability and the residual flight time of the fleet. The presented model is considered an innovative bi-objective mixed-integer linear programming model that describes the flight and maintenance problem. Afterwards, Kozanidis and Skipis (2006) validated the model with a real-world case study, based Hellenic Air Force's fleet, and compared the results to the typical Hellenic Air Force's approach, concluding that this optimization model can improve the fleet's availability.

Teixeira *et al.* (2006) exposed the details of a fleet preventive maintenance scheduling problem, and then, developed several models, using heuristic approaches, a set of *ant-colony* based approaches, to solve it. The presented model has as objective the maximization of fleet's availability, by reducing wasted time between maintenance activities. Moreover, this formulated model is applied to the Brazilian Airforce case study. To sum up, a sensitive analysis was done to understand which approach found the best solution, which was the one grounded in an ant-colony search with specific local search.

2.2 Maintenance Scheduling in Railway Companies

Huisman *et al.* (2005) give a wide overview of state-of-the-art on operations research models and techniques used by passenger railway operators. Planning problems are usually classified by its planning horizon and can be divided in three planning phases, namely: strategic, tactical and operational. Strategic planning takes decisions concerning the number of rolling-stock units that are necessary for the upcoming years, the long-term availability of crew members and line planning for a long-time horizon. Tactical planning is concerned with the timetable structure, which determines the main product of a railway company. For example, in a cyclic timetable, each line has to be operated either cyclically or periodically. Then, operational planning handles the details of the timetable, for instance, the rolling-stock and crew schedules are constructed. Rolling-stock circulation problem allocates rolling-stock units to the trips to be operated. Service to the passengers, efficiency and robustness are concerns of rolling-stock planning. Routing due to maintenance of rolling-stock addresses the maintenance visits to maintenance facilities. These visits of the rolling-stock units to may already be incorporated in the rolling stock circulation problem. Short-term planning is related to minor modifications to the timetable. For each phase, related to different planning horizons, the most relevant planning problems are discussed and some typical models and algorithms to solve them are presented. Besides timetable, rolling stock scheduling and crew scheduling problems, which are well studied in literature and frequently addressed, some less developed topics are discussed, for instance the shunting problem, as local problem.

Giacco, D'Ariano and Pacciarelli (2014) describe a mixed-integer linear-programming formulation, that uses, minimal cost Hamiltonian cycles for integration of a short-term maintenance planning in a rolling stock circulation problem. A given set of services, empty runs and maintenance tasks must be covered with a minimal amount of rolling stock units. The service and maintenance works are paired through the minimal cost Hamiltonian cycles. Computational results for the Italian railway company Trenitalia are presented and compared with the current rolling stock circulation plan. It is showed that the method can reduce considerably the number of trains and empty runs, and so reduce the costs of the company. Improved solutions are obtained through a sequential approach, which first minimizes the number of train units needed to cover the timetable, and then number of empty runs and maintenance actions.

Méchain (2017) addresses the problem of maintenance planning for a Portuguese railway operating company, Fertagus. A mathematical model is formulated first, concerning the various constraints of the company, but viable to be adapted to fit to any company's specifications. Technical constraints associated with the maintenance yard configuration are introduced in the model. The adaptation to the company involved data collecting related to maintenance activity operations in the maintenance yard. A MILP (Mixed-Integer Linear Programming) optimization model is developed, that minimizes the total cost spent on preventive maintenance and adapted to the company context. The model successfully outputs a technical maintenance planning for all the 52 weeks of a given year. Moreover, it outputs for each week, the maintenance activities that need to be performed on each train unit, the line were the maintenance action occurs and the number of spare parts that are required to fulfil the technical planning. In a broad planning perspective, the study lacks an operational planning capable of taking the obtained maintenance technical plan as an input and verify if the solution found is feasible to be implemented within the operations of each week.

Doganay and Bohlin (2010) also propose a solution for the maintenance scheduling problem of Swedish railway operating companies and are actually an inspiration for Méchain's research (2017). The formulation of a mixed integer programming model of the problem, with spare parts optimization is described. An underlying study carried out, which considers a wider overview on maintenance scheduling, out showed that it was often not useful to have a detailed planning more than a few weeks before it actually occurs. However, it is undoubtedly useful to have access to a less detailed maintenance plan and predict wider issues of the operation, such as the workforce required or the necessary number of spare parts for maintenance.

Tréfond et al., (2017) study a rolling-stock planning problem with a robustness perspective for French passenger trains. First, the concept of robustness is discussed. Accordingly, "*a robust roster should resist, limit delay propagation, or be easily recoverable when a "weak disturbance" occurs*" (Tréfond et al., 2017). It is noteworthy that robustness is not considered by modelling data uncertainty. Some indicators are assessed for the evaluation of rolling-stock rosters. Homogenization of turning-times is the chosen method, to absorb potential delays and so introduce robustness to the roster. More specifically, a construction robustness indicator (turning times homogenization indicator) is used. At SNCF company, the reference tool (PRESTO) calculates a solution to the rolling-stock planning problem. It consists of a multi-step approach to cover demand while minimizing operating costs, and to further add maintenance slots to the roster. The paper proposes a method based on an integrated ILP (Integer Linear Programming) model to add robustness to a roster while maintaining low operating costs compared to PRESTO. Tests were carried out to validate the model and verify the relevance of the used construction robustness indicator. A significant improvement in robustness indicators was observed, while maintaining low operating costs and meeting maintenance requirements. The model improves robustness, keeps operating costs low and considers maintenance requirements. However, the maintenance approach in not very in-depth, considering simply the introduction of maintenance slots. Simple constraints were used, for instance, each unit must go to maintenance depot every three days. Furthermore, the model is possible to integrate some other indicators, for instance the minimization of coupling and uncoupling operations.

2.3 Contribution of the Research

Table 1 - Summary of the analysis of the research works on maintenance scheduling

Author(s) (date)	Keywords	Proposed technical	Contribution
Haghani & Shafahi (2002)	Bus maintenance systems, Bus maintenance scheduling	Mixed integer linear programming	Minimize the number of hours spent by buses to their scheduled service for inspection and maximize usage of maintenance facilities
Kozanidis and Skipis (2006)	Flight scheduling, Maintenance based planning	MILP	Maximize the availability of aircraft and residual flight time of a fleet
Teixeira *et al.* (2006)	Preventive maintenance, Scheduling of fleets of vehicles	Ant-Colony algorithm Heuristic approaches	Maximize availability of a fleet by minimizing unused hours between maintenance activities
Huisman *et al.* (2005)	Optimization, Overview on planning problems arising at Netherlands railways, Literature review	Models and algorithms to solve planning problems	State-of-the-art on operations research models and techniques used by passenger railway operators
Méchain (2017)	---	MILP model	Preventive maintenance weekly planning for 52 weeks under maintenance yard constraints
Doganay and Bohlin (2010)	Maintenance planning, Condition based maintenance, Optimization, Railways	Mixed integer programming	Influence of spare parts and working force on preventive maintenance planning
Giacco, D'Ariano and Pacciarelli (2014)	Maintenance, Mixed-Integer Linear-Programming, Railway planning, Rolling stock circulation	MILP Minimal cost Hamiltonian cycles	Minimizing the number of rolling Stock units needed to cover timetable with miniatous and the number of empty runs
Tréfond *et* al. (2017)	Railway management, Rolling-stock planning, Robustness, Optimization	Integrated ILP model	Add robustness to a roster while covering demand and maintaining low operating costs and to further add maintenance slots to the roster

Although there are many studies addressing the problem of scheduling operational activities inherent to railway companies, this research work aims to answer the needs and further research directions raised by Méchain (2017). In that sense, it aims to verify if the solution found is feasible to be implemented within the operations of each week, for an operational planning capable of taking the obtained maintenance technical plan as an input. Thus, a model by Tréfond *et al.* (2017) was chosen to be suitable, if it is properly adapted and further developed in what concerns to its maintenance approach. Mainly for this reason, a more focused and narrowed state of the Art was presented, as no other paper was found suitable to add new contributions to the research work chosen as a basis for the proposed dissertation (Tréfond *et al.*, 2017), and which is quite recent (i.e. from last year).

3 A Mixed-Integer Linear Programming Model

In chapter 3, the optimization model is described and explained in detail. The rolling-stock planning problem is fully defined and all the data, the objective function, the decision variables and the associated constraints are also presented and discussed in detail.

3.1 A Mixed-Integer Linear Programming Model

The present mathematical model is an adaptation of the model presented by Tréfond *et al.* (2017) on the problem of robust rolling-stock planning for French passenger trains (Tréfond *et al.*, 2017). To fit the Fertagus case study and integrate the information associated with the preventive maintenance model extracted from (Méchain, 2017), that model had to be adapted and a considerable part had to be changed.

Part of the rolling-stock planning problem consists in providing the minimal number of rolling-stock (or train) units to cover a specific timetable. (Tréfond *et al.*, 2017) describe a software, used by SNFC company, that computes this minimum number of trains and used it as an input for their model. In the proposed model, this number is also assumed as an input that should be provided by the train operating company. In fact, the use of a robust model aims to reduce the risk of needing an extra train unit to cover possible disturbances in the operation, and so maximize the availability of train units.

Some adjustments and changes were made in the roster construction part. Instead of obtaining a cyclic roster as an output, as in (Tréfond *et al.*, 2017), this model outputs a roster allocated to a given week and regarding its maintenance requirements. Briefly, a cyclic roster is a roster repeated every week, with concern for an equal usage of each train unit. Cyclic, because after a given number of cycles (equals to the number of train units), all units were subjected to the same services, and consequently all units travelled the same number of kilometres. It promotes an equal wear of the fleet, but it was not applied to this case study, because of the differences in the maintenance needs of each week.

The approach made to the maintenance requirements was completely changed, being the major research contribution of the present work. It considers a preventive maintenance planning made for all the 52 weeks of a given year, to build the roster for each week of the year. Ultimately, the model outputs a weekly schedule, contemplating the preventive maintenance actions that must be performed on a given week and with the smallest maintenance cost possible. Therefore, all technical constraints related to maintenance are new constraints and they provide the main research contribution of the present work.

The proposed model is a Mixed-Integer Linear Programming model. It is a decision model capable of using the preventive maintenance planning and the timetable activities from a train operating company to build a rolling-stock planning roster for each week of the year.

The cost components of the objective function are taken from Tréfond et al. (2017), except the last one, which was created to include the maintenance activities in the model. The final decision model uses four decision variables to optimize the costs of the operation, explained in detail further on.

The next sections fully define the model and detail the problem data, the decision variables, the objective function and the constraints.

3.2 The Rolling-stock Planning Problem Definition

A **task T_i** is defined as a non-splittable trip to be realized between one **departure station Sd_i** and one **arrival station Sa_i**. It is also characterized by **departure** and **arrival times, Dd_i** and **Da_i** respectively. The **demand DEM_i**, corresponding to the number of train units needed to perform a task, and the **capacity CAP_i**, corresponding to the maximal number of train units that can be used to cover that task, are also known. This information, related to the tasks that must be performed, is part of a timetable problem, which is not a concern of the present study. It should be provided by the railway operators, usually in cooperation with the infrastructure manager. Figure 1 outlines the information regarding one example task. The set of all tasks that must be performed corresponds to a timetable, which must be provided as an input for the model.

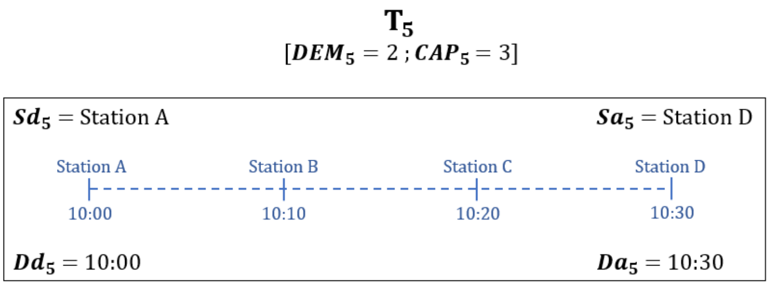

Figure 1 - Example of a task T_5 with all the information that characterizes it

To cover each task, it is possible to provide one or more train units. A **train or rolling-stock unit U_k** is a set of rail coaches that cannot be divided. Two or more units can be coupled to create a multiple unit, so that it can cover a higher demand task. A unit can be assigned to two **successive tasks T_i and T_j** if T_j starts from arrival station of T_i, and if the turning time between the two tasks is greater than a technical threshold (**minimal turning time TM_s**, which is specific to each **station s**). A **turning time** is the time between the arrival time of a task and the departure time of the next task covered by the same unit. More precisely, the turning time between tasks T_i and T_j is equal to $Dd_j - Da_i$. Two tasks possible to merge successively are illustrated in Figure 2.

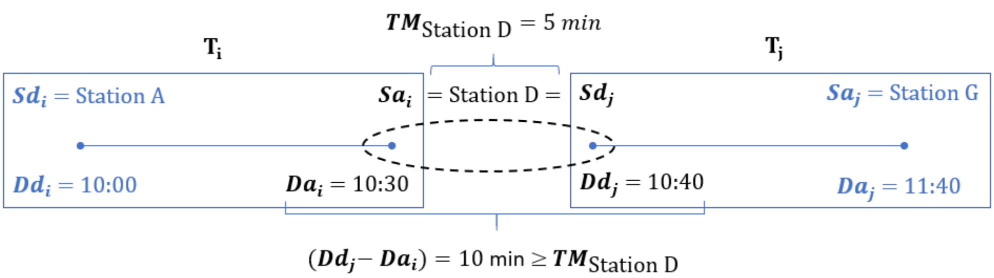

Figure 2 - Two successive tasks T_i and T_j satisfying the merger conditions

For each **time-period p** (e.g. one day or one week) with an associated **length L**, a train unit is assigned to a sequence of tasks called **row of a unit R_k**. Figure 3 shows the example of a row.

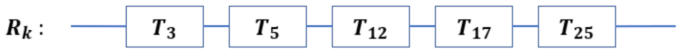

Figure 3 – Example of a row k to be associated with a train unit

A **rolling-stock roster** is a sequence of rows. A unit can accomplish two **successive rows R_k and $R_{k'}$** during **two successive periods p and $p+1$** if R_k follows $R_{k'}$, i.e. if the unit can cover successively the last task of R_k in period p and the first task of $R_{k'}$ in period $p+1$.

A roster composed of NU units is **cyclic** if the rows can be numbered from 1 to NU, so that for all k from 1 to $NU-1$, row R_{k+1} follows row R_k, and row R_1 follows row R_{NU}. Then, on the first period p, each unit U_k can be assigned to row R_k. The period after $p+1$, for all k from 1 to $NU-1$, unit U_k covers row R_{k+1}, and unit U_{NU} covers row R_1.

Figure 4 supports the understanding of the previously explained concepts. As mentioned before, the proposed model is not concerned to output a cyclic roster, but a roster related to a given week (only the orange circle in the figure). Therefore, for each week a different roster is obtained, depending on its maintenance planning. This is a noteworthy modification to the base model.

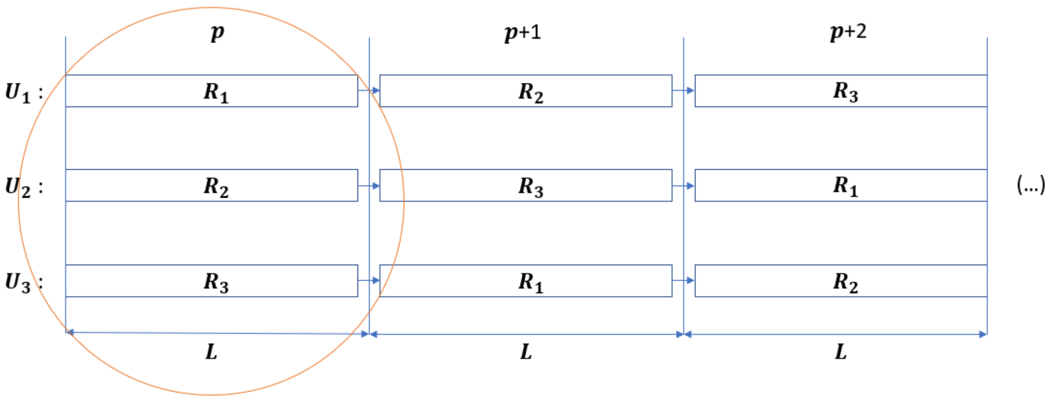

Figure 4 – 3 rows cyclic roster for successive time periods, including a roster associated to a given time-period in the orange circle

In addition, train units must respect maintenance requirements. For that purpose, an "optimal" technical planning (Méchain, 2017) is used as an input to this model. "Optimal" because it reduces the cost of preventive maintenance in the company, presenting the maintenance actions that need to be performed on each week, in each unit. In other words, each unit needs to make scheduled periodic visits to a maintenance site (called depot), without compromising the service/operation tasks that must be repeated every week.

A **maintenance action $KM_{k,m}$** is defined as a preventive maintenance intervention to be realized between two successive tasks T_i and T_j, on a specific unit and at a specific station called depot. There is a limited number of kind of maintenance actions, which can be performed, and each kind of maintenance action has a characteristic **duration** $\boldsymbol{MT_m}$ and **working load** $\boldsymbol{AW_m}$. The working load and the duration relate as follows: $duration = \frac{working\ load}{working\ persons}$. Different kind of maintenance actions require a different number of working persons. Finally, for a given time-period, there is one or more predefined **days d**, when maintenance may occur. The number of days available for maintenance can be less than the number of days of the time-period. For example, when the time-period is one complete week (7 days), the number of days available for maintenance equals 5 if the maintenance workshop (or depot) is closed during the weekend.

Dead-headings are trips with no passengers and can be added to the roster to move units from a station to another. These trips may be necessary to move units to or from the depot to perform maintenance actions, with an associated **duration** $\boldsymbol{DW_{s,depot}}$. Therefore, a unit can be assigned to a maintenance action (programmed in the maintenance plan) between two successive tasks if there is enough time to perform the maintenance action and the necessary dead headings, i.e. if $\boldsymbol{Dd_j} - \boldsymbol{Da_i} \geq \boldsymbol{DW_{s,depot}} +$ $\mathbf{duration}\ \boldsymbol{MT_m} + \boldsymbol{DW_{depot,s'}}$. Figure 5 makes it clear.

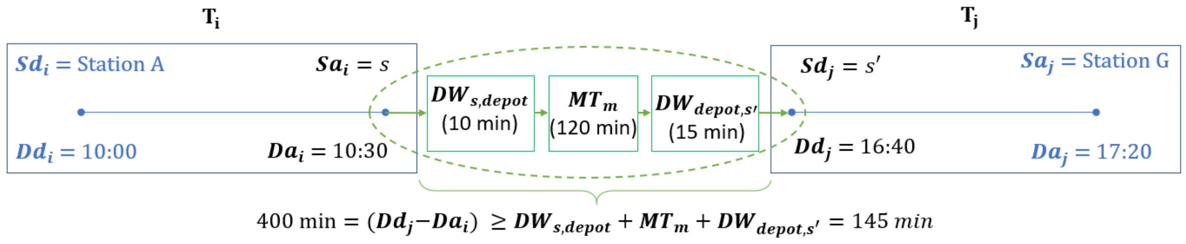

Figure 5 – Maintenance action $\boldsymbol{m}$ and two necessary dead headings satisfying the merger conditions between two successive tasks

Moreover, dead-headings are necessary to cover every task. However, dead-headings cause additional costs for the company and increase the rail traffic in the network. In particular, they are sometimes not possible due to track occupation by other train services. An alternative way to move units consists of using more units than needed to cover a task, i.e. for one task, the units that satisfy the demand are **active**, additional ones are **passive**.

To sum up, a unit is assigned to a row, which is a sequence of tasks, maintenance actions and dead-headings. When the unit is assigned to a task, it is considered either as an active or as a passive unit.

Costs related to a unit are the number of kilometres that it travels. **Active costs** of a unit correspond to the number of kilometres travelled as an active unit, while **passive costs** correspond to the number of kilometres travelled as a passive unit. The total number of units used, and the active costs are called **primary costs**. Costs related to dead-headings and passive costs are called **secondary costs**. **Operating costs** include primary and secondary costs. Both are to be **minimized**. The impact of secondary costs is much lower than the impact of primary costs. However, the present work focusses on the secondary costs minimization, as a rolling-stock circulation planning problem, as the primary costs of the solution to be found remain unchanged.

For a set of tasks and maintenance actions, a feasible solution to this rolling-stock planning problem consists of a roster related to one time-period, in which all tasks and maintenance actions are covered, and technical operating and maintenance constraints are respected. Moreover, the considered problem consists in building a **robust roster**. *"A robust rolling-stock roster should anticipate operational disturbance possibilities in order to limit service quality deterioration and additional costs."* (Tréfond *et al.*, 2017). Improving robustness may be in conflict with operating costs minimization. In practice, it is unacceptable to degrade primary costs, since the obvious solution to improve robustness would be to use more train units (by far the highest cost of the railway operation). As said before, the primary costs are not to be altered. Then, the objective is a trade-off between secondary operating costs (dead-headings and passive units) and robustness, which is quantified by a **robustness indicator** explained further on. Therefore, secondary costs may be deteriorated to build robust solutions, provided that this deterioration is controlled.

The model computes on each task the number of active and passive units and creates dead-headings, so that all tasks are covered, while the operating costs are minimal. The maintenance actions are added, while building the rows of the roster and optimizing its robustness.

3.3 Indexes

k train unit

s station

i task

j task

m maintenance action

d day

3.4 Problem Data

In this subsection, the relevant data to describe the problem is presented in detail and divided up into categories, namely: i) General data; ii) Data related to stations; iii) Data related to tasks; iv) Data related to dead-headings and v) Data related to maintenance. This division aims to facilitate comprehension of the problem data.

General data:

NU number of train units and consequently of roster rows

K set of train units or roster rows, numbered $1..NU$, indexed by k

Data related to stations:

NS number of stations

S set of stations, numbered $1..NS$, indexed by s

TM_s minimal turning time at station s (parameter)

Data related to tasks:

NT number of real tasks to cover

T set of real tasks to cover, numbered $1..NT$, indexed by i, j

Sd_i departure station of task i (parameter)

Sa_i arrival station of task i (parameter)

Dd_i departure time of task i (parameter)

Da_i arrival time of task i (parameter)

DEM_i required number of units to cover task i (parameter)

CAP_i maximal number of units on task i (parameter)

Data related to dead-headings:

$W_{s,s'}$ pairs of stations s and s' between which there can exist a dead-heading (parameter)

$CW_{s,s'}$ length of a dead-heading from station s to station s' in kilometres (parameter)

$DW_{s,s'}$ duration of a dead-heading from station s to station s' in minutes (parameter)

Data related to maintenance:

NM number of maintenance actions

MM set of maintenance actions, numbered $1..NM$, indexed by m

ND number of days for maintenance

D set of days for maintenance, numbered $1..ND$, indexed by d

$KM_{k,m}$ maintenance actions m that need to be performed on each unit k (parameter)

MT_m duration of maintenance action m in minutes (parameter)

AW_m working load of maintenance action m in minutes (parameter)

LN large number (parameter)

3.5 Data Pre-processing

For the modelling of the MILP problem, some new data and identities must be created. Then, an exhaustive explanation is performed for a complete understanding of the model.

BVT set of beginning virtual tasks, numbered $NT + 1..NT + NS$, indexed by i, j

EVT set of ending virtual tasks, numbered $NT + NS + 1..NT + 2 * NS$, indexed by i, j

NTT number of total tasks (real + virtual tasks)

TT set of total tasks, numbered $1..NT + 2 * NS$, indexed by i, j

$R_{i,j}$ processed parameter to identify the set of all pairs of tasks i, j that can be chained up by the same unit

$DdU_{i,j}$ processed parameter for the departure time of a row of a unit

$DaU_{i,j}$ processed parameter for the arrival time of a row of a unit

$\Delta_{k,i,j}$ processed parameter for the turning times homogenization

Virtual tasks, as the name suggests, do not correspond to an actual action. Their function is only to identify the initial and the final stations for each row of a unit. Virtual tasks do not have a demand, a duration nor a capacity. To clarify, **real tasks** are numbered from 1 to NT and the **stations** from 1 to NS; and thus, NS **beginning virtual tasks** are numbered from $NT+1$ to $NT+NS$ corresponding to each station at the beginning of the time-period. Similarly, NS **ending virtual tasks** are numbered from $NT+NS+1$ to $NT+2NS$ corresponding to each station at the end of the time-period. In this model, each unit starts at a station s with a beginning virtual task $NT+s$, executes a sequence of real tasks, and arrives at a station s' with an ending virtual task $NT+NS+s$'.

To build each row of the roster, we first need to identify the **set of all pairs of real or virtual tasks $\boldsymbol{i, j}$ possible to chain up by the same unit**. For this purpose, the variable $\boldsymbol{R_{i,j}}$ is used. More precisely:

$\forall\ (i,j) \in (TT, TT)$,

$$R_{i,j} \begin{cases} = 1 & \text{if the pair of tasks } (i,j) \text{ can be chained up directly by the same unit,} \\ = 0 & \text{otherwise.} \end{cases}$$

The pair of tasks (i,j) can be **chained up directly** by the same unit if stations correspond and, for real tasks, if the turning time between i and j can be respected:

- any pair of real tasks i, j can be chained up if $Sd_j = Sa_i$ and $Dd_j \geq Da_i + TM_{Sa_i}$;
- any real task j can follow a beginning virtual task i, $NT+s$, if $Sd_j = s$;
- any ending virtual task j, $NT+NS+s$, can follow a real task i if $Sa_i = s$.

The parameter $\boldsymbol{W_{s,s'}}$ identifies the **set of all pairs of stations $\boldsymbol{s, s'}$ between which there can exist a dead-heading**. This parameter only presents two values, more precisely, if $W_{s,s'} = 1$ there can exist a dead-heading between s and s', and if $W_{s,s'} = 0$ it is not possible.

The pair of tasks (i,j) can be **chained up** by the same unit **using a dead-heading** if it is possible to insert a dead-heading between the stations that link i and j, and if the duration of the dead-heading respects the turning time between i and j:

- for any pair of real tasks (i,j) it is possible to insert a dead-heading from the arrival station of i to the departure station of j if $W_{Sa_i,Sd_j} = 1$ and $Dd_j \geq Da_i + TM_{Sa_i} + DW_{Sa_i,Sd_j}$;
- for any pair of beginning virtual task i and real task j, $(NT+s, j)$, it is possible to insert a dead-heading from s to the departure station of j if $W_{s,Sd_j} = 1$;
- for any pair of real task i and ending virtual task j, $(i, NT+NS+s)$, it is possible to insert a dead-heading from the arrival station of i to s if $W_{Sa_i,s} = 1$.

The departure and arrival times of a unit have also to be computed. The **departure time of a unit** starting at station s and whose first real task is i is denoted by $DdU_{NT+s,i}$. A unit starts at station s through a beginning virtual task j. Then, it executes a real task i, either directly from station s or from a different station s'. In the latter case, a dead-heading is performed from s to s' with duration $DW_{s,s'}$. Let s' be the departure station of i ($s' = Sd_i$):

- if $s = s'$, then $DdU_{NT+s,i} = Dd_i$ (the unit starts at the same time as task i);
- otherwise, $DdU_{NT+s,i} = Dd_i - DW_{s,s'}$ (the unit starts at the same time as the dead-heading).

Similarly, the **arrival time of a unit** is denoted by $DaU_{i,NT+NS+s'}$. A unit executes a last task i ending at station s. Then, it arrives at station s' through an ending virtual task j, either directly or by performing a dead-heading from s to s' with duration $DW_{s,s'}$. Let s be the arrival station of i $(s = Sa_i)$:

- if $s = s'$, then $DaU_{i,NT+NS+s} = Da_i$ (the unit ends at the same time as task i);
- otherwise, $DaU_{i,NT+NS+s} = Da_i - DW_{s,s'}$ (the unit ends at the same time as the dead-heading).

To integrate robustness in the solution, a robustness indicator is used based on the statement that homogeneous turning times bring robustness to a rolling-stock plan. **Turning times homogenization indicator** $\Delta_{k,i,j}$ will discourage short turning times, and so**,** absorb potential delays.

As explained before, the turning time between two successive tasks i and j equals $Dd_j - Da_i$. By default, all turning times lower than 1 minute are considered as 1-minute-turning times. Conversely, turning times higher than 60 minutes are not considered.

For a turning time between real tasks i and j **chained up directly** by a unit k:

$$\Delta_{k,i,j} \begin{cases} = \frac{1}{\max(1, Dd_j - Da_i)} & if\ Dd_j - Da_i \le 60; \\ = 0 & otherwise. \end{cases} \tag{1}$$

For any pair of real tasks i and j **linked by a dead-heading**, there are two turning times: one between i and $W_{s,s'}$, and one between $W_{s,s'}$ and j. By default, $W_{s,s'}$ is placed in the middle, so that both turning times are equal. So, two equal turning times are considered:

$$\Delta_{k,i,j} = \frac{2}{\max\left(1, \frac{Dd_j - Da_i - DW_{Sa_i,Sd_j}}{2}\right)} \tag{2}$$

3.6 Decision Variables

$\forall\ k \in K,\ \ i \in TT,$

$$x_{k,i} \begin{cases} = 1 & \text{if unit k covers task i;} \\ = 0 & \text{otherwise.} \end{cases}$$

$\forall\ k \in k,\ \ i \in TT,\ \ j \in TT,\ \ (i,j)|R_{i,j} = 1,$

$$y_{k,i,j} \begin{cases} = 1 & \text{if unit k covers successively tasks i and j;} \\ = 0 & \text{otherwise.} \end{cases}$$

$\forall\ k \in K,\ \ i \in TT,\ \ j \in TT,\ \ m\ \in MM,\ \ (i,j)|R_{i,j} = 1,\ \ (k,m)|KM_{k,m} = 1,$

$$yM_{k,i,j,m} \begin{cases} = 1 & \text{if maintenance action m is performed on unit k, between the pair of tasks (i, j);} \\ = 0 & \text{otherwise.} \end{cases}$$

$\forall\ k \in K,\ \ d \in D,$

$$zM_{k,d} \begin{cases} = 1 & \text{if unit k covers any maintenance action on day d;} \\ = 0 & \text{otherwise.} \end{cases}$$

3.7 Objective Function

This MILP model is based on costs of an existing cost-optimal solution, computed to improve robustness. Robustness is considered by optimizing the turning times homogenization robustness indicator. However, the resulting criteria may conflict with operating costs minimization. In practice, it is unacceptable to degrade primary costs, and so, the objective function has to be a trade-off between robustness and secondary costs. It is a weighted sum of three terms related to operating costs, robustness indicator and shuntings for maintenance purpose, as described further on.

Objective function to be minimized:

$$PW * \sum_{k\in K} \sum_{i\in TT|R_{i,j}=1} \sum_{j\in TT|R_{i,j}=1} CW_{Sa_i,Sd_j} * y_{k,i,j} + PTHOM * \sum_{k\in K} \sum_{i\in T|R_{i,j}=1} \sum_{j\in TT|R_{i,j}=1} \Delta_{k,i,j} * y_{k,i,j} + PTZM * \sum_{k\in K} \sum_{d\in D} zM_{k,d} \qquad (3)$$

3.7.1 Secondary Costs

The first term of (3) corresponds to the secondary operating costs. Secondary costs are composed of passive trips and dead-headings. Passive trips are usually negligible compared to dead-headings, and therefore, they are not accounted for in the model. In the objective function, costs related to a dead-heading linking two tasks i and j have a specific penalty, in particular its length CW_{Sa_i,Sd_j}, which is the number of kilometers of a dead-heading between station Sa_i and station Sd_j.

3.7.2 Robustness Indicator

The second term of (3) is the value of the robustness indicator based on turning times. As mentioned before, there is a need to homogenize turning times in the roster, so the turning times homogenization indicator $\Delta_{k,i,j}$ is to be minimized.

3.7.3 Shuntings for Maintenance

The last term of (3) takes into account the number of shuntings to the depot needed to be executed, to fulfil the maintenance actions. It is desirable to run shuntings as lower as possible due to two reasons: On one hand, it is a considerable expense to the company. On the other hand, minimizing the number of shuntings leads to the maximization of the availability of the train units, since they cannot run service tasks while parked at the depot.

3.7.4 Weights of the Objective Function

As described above, the objective function is a weighted sum of three terms. We define the following weights:

PW	weight associated with dead-heading in the objective function
$PTHOM$	weight associated with turning times in the objective function
$PTZM$	weight associated with shuntings for maintenance in the objective function

These parameters have to be set according to a trade-off between robustness and costs. Dead-headings generate the most important costs, then the weight PW should be high enough to limit the increase of corresponding costs. Then, the robustness weight should reflect the trade-off between robustness and costs. Shuntings also generate major costs, then $PTZM$ should be high enough to avoid more shuntings to the depot than necessary.

3.8 Constraints

To implement the various specifications of the model, the objective function presented in the previous chapter must be subjected to a few constraints.

3.8.1 Existence of a Roster

The existence of a rolling-stock roster of NU units without maintenance requires the verification of the following constraints:

$$\sum_{i \in BVT} x_{k,i} = 1 \quad \forall\ k \in K \tag{4}$$

$$\sum_{j \in TT | R_{i,j}=1} y_{k,j,i} = \sum_{j \in TT | R_{i,j}=1} y_{k,i,j} \quad \forall\ k \in K, i \in T \tag{5}$$

$$\sum_{k \in K} x_{k,i} \geq DEM_i \quad \forall\ i \in T \tag{6}$$

$$\sum_{k \in K} x_{k,i} \leq CAP_i \quad \forall\ i \in T \tag{7}$$

$$x_{k,i} = \sum_{j \in TT | R_{i,j}=1} y_{k,i,j} \quad \forall\ k \in K, i \in T \cup BVT \tag{8}$$

$$x_{k,i} = \sum_{j \in TT | R_{i,j}=1} y_{k,j,i} \quad \forall\ k \in K, i \in EVT \tag{9}$$

Constraints (4) guarantee that any unit starts with a beginning virtual task. Constraints (5) assure spatio-temporal coherence. A unit assigned to a task i, which arrives at station Sa_i, can either be assigned to a next task j, whose departure station $Sd_j = Sa_i$, or it can stay at station Sa_i. In the latter case, its next task will be an ending virtual task. This is modelled by the following formulation: for any real task i and any unit k, if there exists a task $j1$ so that unit k chains up $j1$ and i, then there exists a task $j2$ so that a unit k chains up i and $j2$.

According to constraints (6), a real task i must be covered by at least DEM_i units. Constraints (7) assure that at most CAP_i units cover i.

Constraints (8) express variables $x_{k,i}$ depending on the variables $y_{k,i,j}$ for any real or beginning virtual task i. Ending virtual tasks do not have successors. Then, constraints (9) define variables $x_{k,i}$ for each ending virtual task i.

Since this study is based on an existing solution, the station capacity constraint is already guaranteed. Then, we do not need to consider it in the ILP model.

3.8.2 Maintenance

Regarding the optimal technical planning that is used as an input to this model and the related maintenance actions that need to be inserted in the pairs of service tasks, the following constraints were formulated to encompass the planned maintenance actions:

$$yM_{k,i,j,m} \leq y_{k,i,j} \quad \forall\ k \in K,\ i \in TT,\ j\ \in TT,\ m\ \in MM \mid R_{i,j} = 1\ \wedge KM_{k,m} = 1 \tag{10}$$

$$yM_{k,i,j,m} * \left(Dd_j - Da_i - DW_{Sa_i,depot} - DW_{depot,Sd_j}\right) \geq \textstyle\sum_{m1 \in MM} yM_{k,i,j,m1} * MT_{m1} + 5 * \left(\left(\sum_{m1 \in MM} yM_{k,i,j,m1}\right) - 1\right) \quad \forall\ k \in K,\ i \in TT,\ j\ \in TT,\ m\ \in MM\ |KM_{k,m} = 1\ \wedge\ R_{i,j} = 1 \tag{11}$$

$$\textstyle\sum_{i \in T} \sum_{j \in T} \sum_{d \in D} yM_{k,i,j,m} = KM_{k,m}\ \ |R_{i,j} = 1 \wedge Da_i + DW_{Sa_i,depot} \geq 9 * 60 + (d-1) * 24 * 60 \wedge Dd_j - DW_{depot,Sd_j} \leq 18 * 60 + (d-1) * 24 * 60 \quad \forall\ k \in K,\ m\ \in MM\ |KM_{k,m} = 1 \tag{12}$$

$$\textstyle\sum_{i \in TT} \sum_{j \in TT} yM_{k,i,j,m} = 0\ \ |i > NT \vee j > NT \quad \forall\ k \in K,\ m\ \in MM\ |KM_{k,m} = 1 \tag{13}$$

$$\textstyle\sum_{k \in K} \sum_{m \in MM} AW_m * yM_{k,i,j,m} \leq 5 * 8 * 60\ \ |KM_{k,m} = 1 \quad \forall\ i \in T,\ j\ \in T,\ d\ \in D\ \ |R_{i,j} = 1 \wedge Da_i + DW_{Sa_i,depot} \geq 9 * 60 + (d-1) * 24 * 60 \wedge Dd_j - DW_{depot,Sd_j} \leq 18 * 60 + (d-1) * 24 * 60 \tag{14}$$

$$\textstyle\sum_{m \in MM} \sum_{i \in T} \sum_{j \in T}\ yM_{k,i,j,m} \leq zM_{k,d} * LN\ \ |Da_i + DW_{Sa_i,depot} \leq d * 24 * 60\ \wedge Da_i + DW_{Sa_i,depot} \geq (d-1) * 24 * 60 \wedge Dd_j - DW_{depot,Sd_j} \leq d * 24 * 60 \wedge Dd_j - DW_{depot,Sd_j} \geq (d-1) * 24 * 60 \quad \forall\ k \in K,\ d\ \in D \tag{15}$$

$$\boldsymbol{yM_{k,i,j,m} = 0 \quad \forall\ k \in K,\ i \in TT,\ j\ \in TT,\ m\ \in MM\ |R_{i,j} = 0 \vee KM_{k,m} = 0} \tag{16}$$

Constraints (10) guarantee coherence between each pair of tasks that is performed and the associated maintenance actions. In other words, a unit k covering a maintenance action m between the pair of tasks (i, j) also covers (i, j).

Constraints (11) express that for a train unit k, the amount of time spent on the various (or single) maintenance actions $m1$, which are performed between the pair of tasks (i,j), cannot exceed the amount of time indeed available for those maintenance actions. The time spent on dead headings to the depot is accounted for. It is assumed that only one maintenance action can be performed at a time on the same unit and a 5-minutes interval of change between two consecutive maintenance actions.

Constraints (12) assure that a maintenance action m associated with a train unit k will only be performed, if it was previously introduced in the technical plan, and also forces a maintenance action that is in the plan to be realized.

Constraints (13) forbid a maintenance action to occur after a beginning virtual task or before an ending virtual task. Otherwise, the purpose of the virtual tasks would not be respected.

Constraints (14) ensure that the sum of working loads AW_m related to all maintenance actions to be performed on a given day does not exceed the maximum working load available for one day of work: 5 men working 8 hours per day. Furthermore, it forces units to arrive and leave the depot within the operating hours of the workers (between 9:00 and 16:00). The goal is to maximize the availability of units. A unit parked in the depot without benefitting from any maintenance action implies a reduction of the resources available.

Constraints (15) assure that if there is a maintenance action on a given day d and a given unit k, the variable $zM_{k,d}$, relative to a specific unit and day cannot be zero. In other words, it assures a coherence between the variables $yM_{k,i,j,m}$ and $zM_{k,d}$.

Constraints (16) guarantee that if two tasks i and j cannot be chained or if a maintenance action m associated with a train unit k was not previously introduced in the technical plan, the variable $yM_{k,i,j,m}$ must be zero.

3.8.3 Decision Variables

$$\boldsymbol{x_{k,i}} \in \{\mathbf{0},\mathbf{1}\} \quad \forall\, \boldsymbol{k} \in \boldsymbol{K},\ \boldsymbol{i} \in \boldsymbol{TT} \tag{17}$$

$$\boldsymbol{y_{k,i,j}} \in \{\mathbf{0},\mathbf{1}\} \quad \forall\, \boldsymbol{k} \in \boldsymbol{K},\ (\boldsymbol{i},\boldsymbol{j})|\boldsymbol{R_{i,j}} = \mathbf{1} \tag{18}$$

$$\boldsymbol{yM_{k,i,j,m}} \in \{\mathbf{0},\mathbf{1}\} \quad \forall\, \boldsymbol{k} \in \boldsymbol{K},\ \boldsymbol{i} \in \boldsymbol{TT},\ \boldsymbol{j} \in \boldsymbol{TT},\ \boldsymbol{m} \in \boldsymbol{MM} \tag{19}$$

$$\boldsymbol{zM_{k,d}} \in \{\mathbf{0},\mathbf{1}\} \quad \forall\, \boldsymbol{k} \in \boldsymbol{K}, \boldsymbol{d} \in \boldsymbol{D} \tag{20}$$

The variables relative to constraints (17), (18), (19) and (20) are all binary variables.

4 Application – Model Implementation and Validation

In chapter 4, an illustrative example is presented in detail to provide a better understanding of the concepts introduced in the previous chapter. Through this implementation it is intended to test and validate the model. In the first section, the implementation in FICO Xpress is described. In the second section, the illustrative example and its parameters are defined, and, in the third section, the results of the mathematical model are analysed.

4.1 Model Implementation in FICO Xpress Optimization Software

Optimization is nowadays a powerful tool to seek for best solutions, both for costs reduction and performance improvement. Technological advancements lead to an increased computational capacity, which allows for formulation and solution of more complex problems. *"The ever-growing realm of applications and the explosion in computing power is driving optimization research in new and exciting directions."* (Nocedal and Wright, 2011). *"Every year optimization algorithms are being called on to handle problems that are much larger and complex than in the past."* (Nocedal and Wright, 2011).

Indeed, it is a very significant resource for railway companies, as it enables for very refined solutions in an industry where the variables are multiple and the capacity for a fast update of solutions is a strategic advantage. The capacity to obtain reduced cost solutions stands out between the advantages of using optimization techniques, due to the high operational and maintenance costs inherent to the railway operation. Therefore, optimization solvers can have a major impact on cost savings. An optimized solution is achieved either by maximizing or minimizing at least one function, the objective function. For this purpose, several softwares with optimization solvers can be found in the market, such as: Excel, Gurobi, IBM CPlex or FICO Xpress.

For the current study, FICO Xpress software was chosen. "*FICO Xpress Optimization allows businesses to solve their toughest problems, faster.*" (Fico, 2018). It provides different kind of solvers for the mathematical programming model: quadratic solvers, nonlinear solvers, mixed-integer linear solvers. "FICO's powerful and versatile algorithms solve for large-scale, linear and mixed integer problems, as well as non-linear problems." (Fico, 2018). In the current proposal, the objective function is linear and accordingly, the constraints are linear as well. The decision variables are integers. Therefore, a mixed-integer linear solver was selected among the ones available in the software.

Chosen the solver, the mathematical formulation was written in the program's specific language, Mosel. The algorithm is grouped in several parts. In the declarations, the constants, parameters, sets and decision variables are defined. In the initializations, the information that needs to be read from data files is designated and the related data files assigned. It is clearer to present the information in various data files, grouped by themes, since the case study has a considerable size. Otherwise, the program would get confusing due to an excess of information present in the command window. Furthermore, it is much easier to write the input data in excel files and then convert them into data files (which can then be read by FICO Xpress). Also, in case there is a need to alter the input data, it can be solved by simply copying and pasting the new contents of the excel file in the data file. In the pre-processing block, the conditions for the processing of certain parameters are defined. These parameters are sets calculated from relations between other parameters and constants, which are direct inputs to the model. For that reason, they are called processed parameters. Then, the objective function to be minimized by the solver and its related constraints are added. At last, the programs outputs are defined. This information, to be provided to the user, is in part displayed in the output window of FICO Xpress and the rest written in a data file created for that purpose. Instead of simply presenting the results for the decision variables that define the rolling-stock plan ($y_{k,i,j}$, $yM_{k,i,j,m}$, $zM_{k,d}$), which would require a huge effort for the interpretation of the results, the program creates a result data file, that shows the rolling-stock plan in a user-friendly manner. This data file is the ultimate output.

Therefore, for each train unit at a time three groups of information are displayed. From the values of the decision variable $y_{k,i,j}$ that equal one (unit k covers successively tasks i and j) it is said: "Pair of tasks $[i-j]$ was performed". Thus, all service tasks to be covered by a specific unit are presented. Furthermore, if also the parameter $CW_{s,s'}$ (length of a dead-heading from station s to station s') is bigger than zero, or in other words, a dead-heading must be performed to link task i to j, it is said: "A dead heading was performed between tasks $[i-j]$". So, the dead-headings to be performed are given and when they need to be performed. From the values of the decision variable $yM_{k,i,j,m}$ that equal one (maintenance action m is performed on unit k, between the pair of tasks (i,j)), it is said: "A maintenance slot of kind $[m]$ was covered between tasks $[i-j]$". Thus, the kind of maintenance actions that must be performed by a specific unit are given when they need to be performed. Finally, from the values of the decision variable $zM_{k,d}$ that equal one (unit k covers any maintenance action on day d), it is said: "This unit went to the depot for maintenance on day $[d]$", adding the information regarding the days a unit must go to the workshops in the PMC is displayed. Altogether, the schedule for the train fleet is provided.

4.2 Illustrative Example

The presented illustrative example is a sample of a real problem, but on a smaller scale. It will analyze and solve a small-sized problem representative of the case study.

First, “toy examples” were used to test the model, through a careful analysis of their corresponding solutions. These “toy examples” are designed to be as simple as possible, so that they can test a specific property of the model and validate that property. The elaborated illustrative example was inspired on the various “toy examples” and thoroughly conceived to present the main aspects of the model, in a scale size that allows for an easy comprehension of the model.

In the current example, 3 train units have to cover 5 tasks and one of them has to go to the depot to perform 2 maintenance actions in a time-period of 1 day. 3 kinds of maintenance actions are considered, with an associated duration and amount of work. Dead-headings must be used to cover all tasks and satisfy maintenance requirements.

The goal is to find the best feasible solution that outputs a rolling-stock planning to the given time-period. For the current model, the best feasible solution means the lowest cost solution that covers every task and performs every maintenance action present in the technical plan.

Tables 2 to 9 provide values for the parameters used in the mathematical model relative to the illustrative example.

Table 2 – Information concerning stations

Station Name	Station Number, s	Minimal Turning Time, TM_s (min)
Roma-Areeiro	1	1
Pragal	2	1
PMC (depot)	3	1
Setúbal	4	1

In Table 2, the first column gives the stations name, the next one their corresponding number and the last column their associated minimal turning time (in minutes). Roma-Areeiro, Pragal and Setúbal are the stations where the service tasks can start and end, or in other words, where there is an entrance and exit of passengers. PMC is the depot station, where only empty trains (without passengers) can enter to perform maintenance.

Table 3 - Pairs of stations between which there can exist dead-headings.

$W_{s,s'}$		s'			
		1	**2**	**3**	**4**
s	**1**	0	1	1	1
	2	1	0	0	0
	3	1	0	0	1
	4	1	0	1	0

In Table 3, s and s' are respectively the departure and arrival stations of a possible dead-heading. If the value of $W_{s,s'}$ equals zero, a dead-heading between stations s and s' is not possible. Otherwise, its value would be equal to one. The only station that presents constraints relative to dead-headings is station 2 (Pragal). Train units can only link Pragal through a dead-heading to Roma-Areeiro. The reasons for these kinds of restrictions are related to infrastructure and operation management, and their discussion and analysis are considered outside the scope of the present study.

Table 4 - Length of dead-headings**.**

$CW_{s,s'}$ (km)		s'			
		1	**2**	**3**	**4**
s	**1**	0	11,68	25,6	54,16
	2	11,68	0	0	42,47
	3	25,6	0	0	28,6
	4	54,16	42,47	28,6	0

Table 4 shows the distance in kilometres between stations s and s'. $CW_{s,s'}$ is set as zero if there is no information on that distance and only if a dead-heading between stations s and s' is not possible. Nevertheless, the opposite is not necessarily true.

Table 5 - Duration of dead-headings**.**

$DW_{s,s'}$ (min)		s'			
		1	**2**	**3**	**4**
s	**1**	0	16	24	45
	2	16	0	0	0
	3	24	0	0	21
	4	45	0	21	0

Table 5 shows the duration of a dead-heading between stations s and s' (in minutes). $DW_{s,s'}$ is set to zero if there is no information on the duration and only if case a dead-heading between stations s and s' is not possible. Nevertheless, the opposite is not necessarily true.

Table 6 – Constants used.

Constant	**Unit**	**Value**
NU	---	3
NS	---	4
NT	---	5
ND	day	1
NM	---	3
LN	---	10000
PW	---	1500
$PTHOM$	---	300
$PTZM$	---	200

In Table 6, all the constants used in the example are shown, by order: the number of train units (and consequently of roster rows), the number of stations and the number of real tasks. Then, the number of days available for maintenance, which can be less than the number of days of the time-period. Still, in the present example the time-period and the number of days available for maintenance are equal (1 day). Then, the number of maintenance actions that can be performed in the depot. LN is a large number to be used in one of the constraints regarding maintenance, and it is not directly related to the values in the example. Finally, the weights of the different terms of the objective function: the weight associated with dead-headings, the weight associated with turning times and the weight associated with shuntings for maintenance purpose.

Table 7 - Information about tasks

Task (T_i)	DEM_i	CAP_i	Sd_i	Sa_i	Dd_i (min)	Da_i (min)
1	2	2	1	4	545	603
2	1	1	4	1	610	668
3	1	1	1	2	663	680
4	1	2	1	4	565	623
5	1	1	4	1	797	855
6	0	0	1	1	0	0
7	0	0	2	2	0	0
8	0	0	3	3	0	0
9	0	0	4	4	0	0
10	0	0	1	1	0	0
11	0	0	2	2	0	0
12	0	0	3	3	0	0
13	0	0	4	4	0	0

In Table 7, the first column identifies the various tasks. The next columns give the required number of units, maximal number of units, departure station, arrival station, departure time and arrival time of a task. Tasks 1 to 5 are real tasks. Tasks 6 to 13 are virtual tasks, and for that reason only have a departure and arrival station (the other values are zero). The departure and arrival times are in minutes. The corresponding time in hours for the departure time of the first task is 9h05 and for the arrival time of the last task is 14h25. Some tasks may occur at the same time, and for that reason, they cannot be covered by the same unit. For example, task 3 starts before the conclusion of task 2. Some tasks may have to be covered by more than one unit. For example, task 1 must be covered exactly by 2 units (no more and no less). In opposition, task 2 must be covered by one unit. Task 4, on the other hand, can be covered either by one or two train units. If covered by 2 units one of them is considered a passive unit.

Table 8 - Maintenance actions that need to be performed on the given planning horizon

$KM_{k,m}$		m = 1	2	3
k	1	0	0	0
	2	0	1	1
	3	0	0	0

In Table 8, k and m are respectively the train units and maintenance actions. $KM_{k,m}$ equals one when a maintenance action must be performed on a specific unit. Specifically, unit 2 must perform two kinds of maintenance actions: 2 and 3. The other units have no planned maintenance actions for the given time-period. Therefore, unit 2 must go to the depot at least once to satisfy the maintenance requirements. This information is provided by a technical planning regarding preventive maintenance (i.e. it is considered an input).

Table 9 – Duration and working load of maintenance actions

Maintenance Action, m	**MT_m (min)**	**AW_m (min)**
1	186	744
2	53	210
3	60	60

In Table 9, the first column identifies the different kinds of maintenance actions. The second column exhibits their duration and the third column their working load. Since the working load and the duration have a known relation ($duration = \frac{working\ load}{working\ persons}$), it can be concluded that maintenance actions 1 and 2 are performed by 4 workers and maintenance action 3 by only 1 worker. The maximum number of working persons available considered in the model is 5.

4.3 Results of the Optimization Model for the Illustrative Example

Once the algorithm converges to a solution, the program displays the minimum cost found for the planning horizon (i.e. over the given 1-day time period). Figure 6 shows the minimum cost obtained for the illustrative example. It also creates a data file (Figure 7) with the information concerning the pairs of tasks that were covered by each unit, the dead-headings that were performed and finally, the maintenance actions that were executed and the days a unit spent in the depot for those maintenance actions. The displayed information enables the rolling-stock planning for the given time period. It should be noted that in case a solution cannot be found, because not every task or maintenance action is covered, the program will show no results.

```
The minimum cost is : 81562.9
Begin running model
End running model
```

Figure 6 - Minimum cost obtained for the illustrative example

```
Train Unit #1
Pair of tasks [1-2] was performed
Pair of tasks [2-10] was performed
Pair of tasks [6-1] was performed

Train Unit #2
Pair of tasks [4-5] was performed
Pair of tasks [5-10] was performed
Pair of tasks [6-4] was performed
A maintenance slot of kind [2] was covered between tasks [4-5]
A maintenance slot of kind [3] was covered between tasks [4-5]
This unit went to the depot for maintenance on day [1]

Train Unit #3
Pair of tasks [1-3] was performed
Pair of tasks [3-11] was performed
Pair of tasks [6-1] was performed
A dead heading was performed between tasks [1-3]
```

Figure 7 – Resuls for the rolling-stock planning displayed by the data file

For the sake of comprehensibility, the obtained rolling-stock planning is outlined in Figure 8.

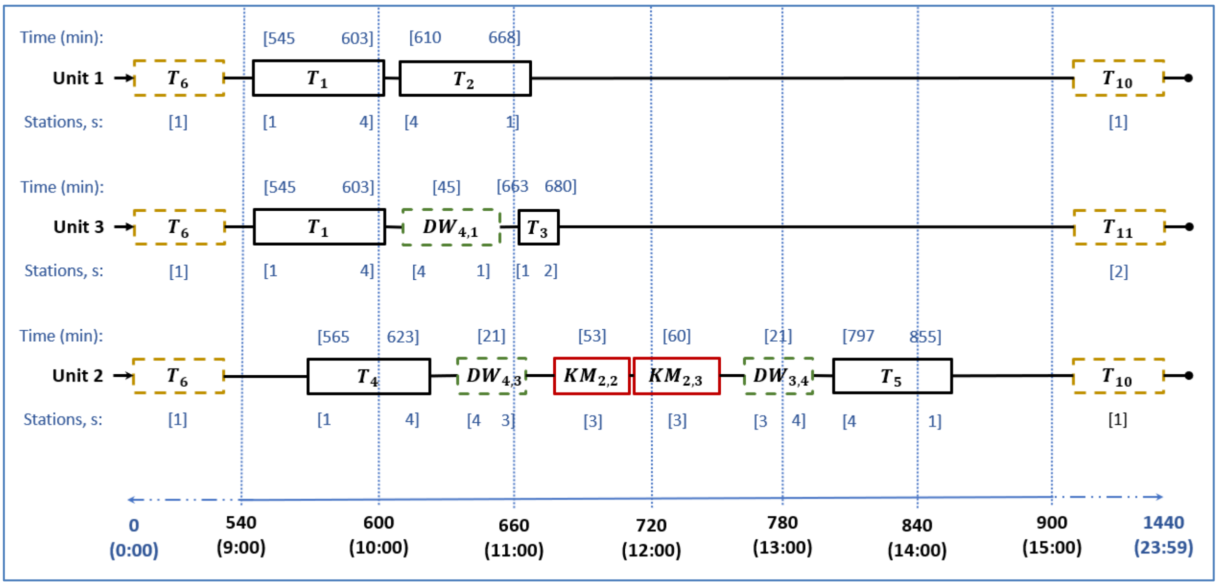

Figure 8 – 3-row-roster to cover timetable demand of Table 7 and maintenance requirements of Table 8, including: virtual tasks (yellow dashed rectangle), real tasks (black rectangle), dead-headings (green dashed rectangle) and mainenace actions (red rectangle)

Several facts should be highlighted. First, we can notice that every task and maintenance action was successfully covered, with the need to make use of dead-headings.

Train unit 1 (U_1) performs two tasks: T_1 and T_2. The pairs of tasks $(6, 1)$ and $(2, 10)$ are beginning and ending virtual tasks, respectively and indicate that the row assigned to U_1 starts and finishes at station 1 (Roma-Areeiro). The link between T_1 and T_2 respects the minimal turning time of station 4 ($TM_4 =$

$1\ min$), as there is a 7-minute gap between the arrival time of task 1 ($Da_1 = 603\ min$) and the departure time of task 2 ($Dd_2 = 610\ min$).

As task T_1 must be covered by two train units, train unit 3 also covers T_1. Furthermore, train unit U_3 also covers T_3. The pairs of tasks $(6, 1)$ and $(3, 11)$ are the beginning and ending virtual tasks, respectively. In this case, row 3 (R_3) also starts at station 1 but ends at station 2 (Pragal). To link tasks T_1 to T_3, a dead-heading must be introduced, as the arrival station of T_1 is Setúbal ($Sa_1 = 4$) and the departure station of T_3 is Roma-Areeiro ($Sd_3 = 1$). This dead-headind has a duration of 45 minutes, so it fits between T_1 and T_3, as $Dd_3 - Da_1 \geq \mathrm{DW}_{4,1}$, i.e. $663\ - 603 = 60\ \geq 45\ min$.

Train unit U_2 performs tasks T_4 and T_5. Again, the pair of tasks $(6{,}4)$ and $(5{,}10)$ are virtual tasks, indicating the stations where R_2 beginns and ends. Two maintenance slots were added between the two tasks: $KM_{2,2}$ and $KM_{2,3}$. Both are performed on the only day available ($d = 1$). Maintenance $m = 2$ has a duration of 53 minutes ($MT_2 = 53\ min$) and maintenance $m = 3$ has a duration of 60 minutes ($MT_3 = 60\ min$). Since they are performed on the same day, they must respect a 5-minute minimal interval between each other. It means that the two maintenance actions take 118 minutes $(53 + 5 + 60)$ to be performed in the depot. Furthermore, the train unit must perform an empty run from the arrival station of T_4 to the depot ($\mathrm{W}_{4,3}$) and another empty run from the depot to the departure station of T_5 ($\mathrm{W}_{3,4}$). Both empty runs have a duration of 21 minutes, so 42 minutes are necessary to move train unit 2 for maintenance purposes. Therefore, a total of 160 minutes $(42 + 118)$ are necessary to perform the maintenance actions, and it fits between tasks T_4 and T_5, as $Dd_5 - Da_4 = 797 - 623 = 174 \geq 160\ min$. Finally, the working loads of the maintenance actions are: $AW_2 = 210\ min$ and $AW_3 = 60\ min$, which summed up are equal to 270 minutes of work needed.

As Figure 6 shows, the minimum cost is 81562,9. A study was carried out to understand the relevance of each term of the objective function (Table 10 - **Costs of the objective** Table 10). Secondary costs have by far the biggest impact (99,6%). The second and third term represent respectively 0,15% and 0,25% of the costs. The weights of each term were not tuned for the illustrative example, although a discussion is provided later on in the next chapter.

Table 10 - Costs of the objective function

1st term (Secondary Costs)	2nd term (Robustness Indicator)	3rd term (Shunting for Maintenance)
81240	122,86	200

For this small-scale example, it took less than a tenth of a second for the model to get to a null gap (Figure 9), i.e. to get an optimal solution. Of course, the computational time will increase if the number of variables also increases (e.g. number of train units, number of stations, number of real tasks, number of maintenance actions and number of days). Figure 9 is an output from FICO Xpress software, which shows useful information such as the size of the matrix before and after the pre-solving stage, as well as information on the final solution.

Table 11 shows how the columns of the matrix are related with the size of the decision variables. All decision variables have a size, which is the product of the size of the sets of their corresponding indices. In total, the matrix is made of 2070 columns before the pre-solving stage. This stage enables to supress some of the redundancy within the initial matrix, which reduces its size. The pre-solving process is specific to FICO Xpress and is not accessible by the user.

As it can be read from Figure 9, the best bound of the objective function corresponds to the best solution. As the objective is to find a minimum, it means that the best bound corresponds to the best lower bound. In the illustrative example, it is interesting to underline that the solution is optimal because the gap is zero. The gap is indeed the ratio (converted in percentage) of the difference between the solution of the total cost and the best bound of the cost: $gap = \frac{total\ cost\ solution - lower\ bound}{\text{total cost solution}} * 100\%$

As said before, the computational time of the current example was very short. However, in larger size problems the computational time will increase, and it may become necessary to stop the computation before the best bound possible is found. The optimality gap is a good indicator of the optimality of the solution when the computation is stopped. The smaller the optimality gap, the closer the solution is to the optimal value.

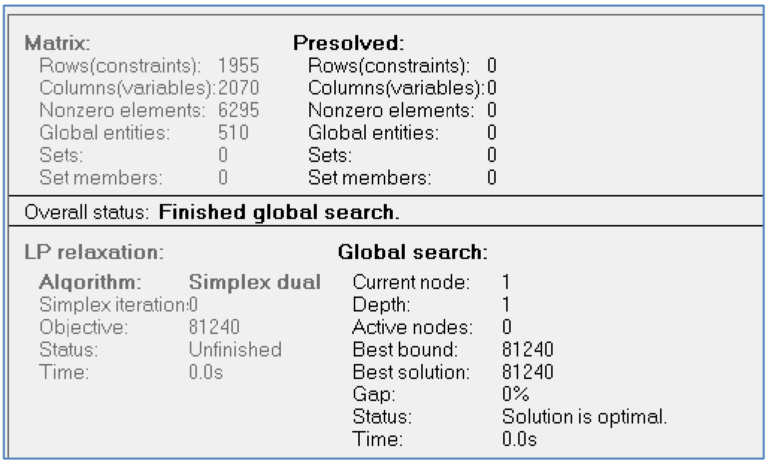

Figure 9 – Model run information

Table 11 – Calculus of the matrix column

Set	Size	Decision Variables	Size	Size of the Columns
$k = \{1, \ldots, 3\}$	3	$x_{k,i}$	$3 * 13 = 39$	$39 + 507 + 1521 + 3 = 2070$
$i = \{1, \ldots, 13\}$	13	$y_{k,i,j}$	$3 * 13 * 13 = 507$	
$j = \{1, \ldots, 13\}$	13	$yM_{k,i,j,m}$	$3 * 13 * 13 * 3 = 1521$	
$m = \{1, \ldots, 3\}$	3	$zM_{k,d}$	$3 * 1 = 3$	
$d = \{1\}$	1			

5 Case Study – Fertagus

In chapter 5, the Fertagus railway operating company is briefly described and the case study specifications are presented. The parameters of the mathematical model are displayed, similarly to the previous chapter.

5.1 Fertagus Train Operating Company

Fertagus is a Portuguese train operating company, which is part of Grupo Barraqueiro, and became the first private train operator to guarantee the commercial concession of a railway line in Portugal. To ensure this suburban passenger transportation concession, it was necessary to win an international public tender for the exploration of the *“Eixo Ferroviário Norte/Sul*”, according to several evaluation criteria, such as the financial model, the travelling times requirements or the proposal tariffs. This concession includes not only the operation of the railway line, but also, the safety and maintenance of the trains and the maintenance of some railway stations, such as several ones from the south bank of the Tejo river.

The railway line is constituted by 14 railway stations, from Roma-Areeiro to Sétubal, as Figure 10 displays, in a route that is realized in about 57 minutes, along 54 kilometres of line and through 3 different lines: *“Linha de Cintura”*, *“Linha do Sul”* and *“Linha do Sado”.* During the route, the train crosses *“25 de Abril”* bridge (Figure 11) through a railway line, in a course of 7 minutes. The railway line, where Fertagus’ trains operates, is also shared with other train operating companies, for instance CP (Comboios de Portugal), and therefore its maintenance cost is responsibility of the infrastructure manager, Infraestruturas de Portugal (IP). Sharing the railway line can result in problems for the railway line and for trains because not every train has the same requirements.

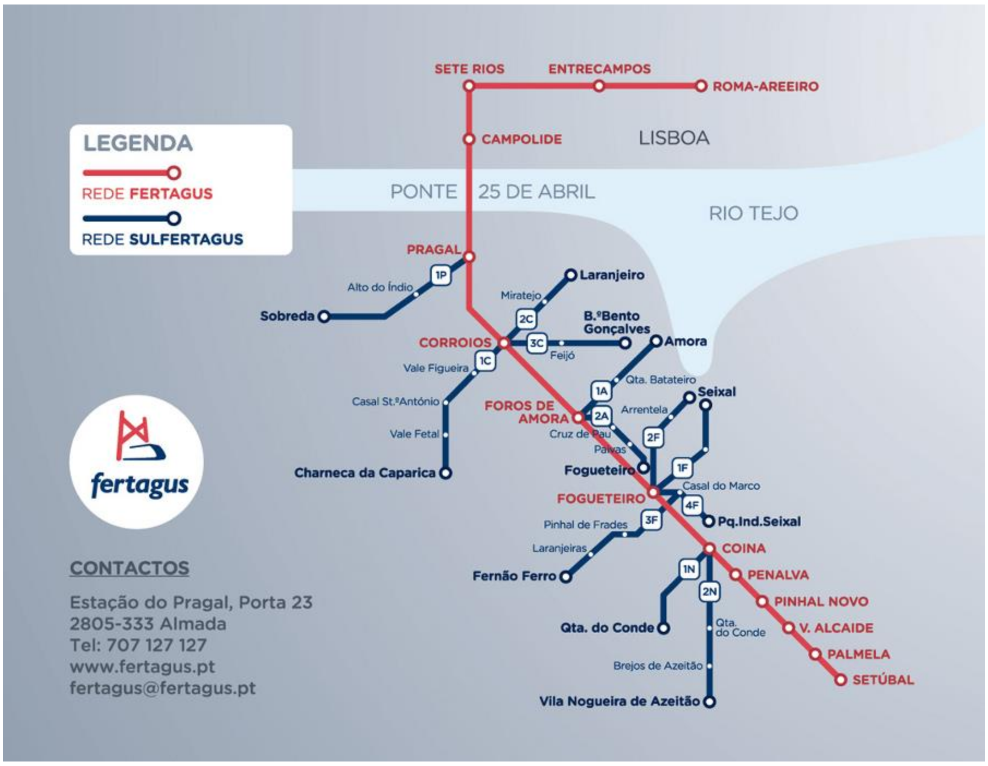

Figure 10 - Fertagus' railway line

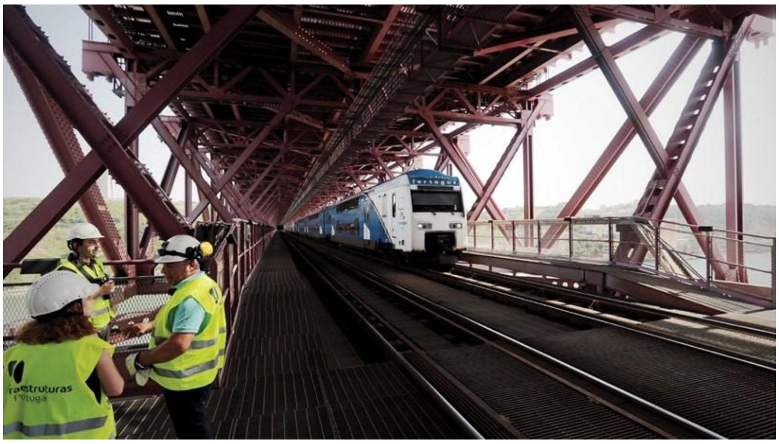

Figure 11 - Railway line across "25 de Abril" bridge

Fertagus has 18 operating trains in its fleet, however, just 17 are used to satisfy the scheduled services. The concession contract has an availability clause, which obliges Fertagus to ensure that no service is affected due to a lack of trains available. The penalty to fail the contract is quite high. For this reason, there are always less then seventeen trains in service, and rotatively, one train is in the PMC.

The maintenance yard was designed to have twelve different railway lines, with different utilities, however just ten were built and are currently in use. There are two lines for mechanical tests, four lines just to park the trains, one line for cleaning and conservation issues and three rooftop-covered lines, which are specific for maintenance activities.

Fertagus is proud of its contribution to improve the quality of life of its users, providing services to about 70,000 passengers a day. One of the great challenges for Fertagus, in the near future, is not only to demonstrate the reliability and feasibility of a public transport such as rail transport, but also to optimize its service and maintenance operations, in order to reduce its costs.

5.2 Input Parameters

During this work, several meetings were scheduled with Fertagus' maintenance planners, to fully understand the details and constraints of the operation that are necessary to model the present case study and also to collect input data for the model. The PMC was visited, and trips were made in Fertagus trains between Roma-Areeiro and Coina. Some additional information was also given by email and phone calls.

In the current case study, the 17 train units that are available for operational services are intended to cover a set of tasks and preventive maintenance actions for the 5 working days of a week (i.e., Monday, Tuesday, Wednesday, Thursday and Friday). The weekend was not considered, as in a regular week the workshop only performs preventive maintenance actions during the working days. Regular, because there are exceptional weeks, as explained further on.

Although the research work by Méchain (2017) outputs a technical plan for 16 kinds of maintenance actions, only 14 are considered in the present research. The reason is that 2 of the 16 maintenance actions are meant to be performed during the weekend in specific periods of the year (exceptional weeks). More specifically, the *"Visita Elétrica"*, which consists of an exhaustive electrical system check-up and is performed once a year before winter, and the *"Torneamento dos Rodados"*, which consists of wheelsets turning and is performed every 120,000 kilometers. The constraints relative to the 14 maintenance actions of the present work are adapted to the working conditions during the week and do not fit the constraints related to the two maintenance actions that are performed during the weekend. Having said that, the integration of Saturday and Sunday in the rolling stock plan, as well as the related maintenance constraints, is left for future research.

The final goal is to find the best feasible solution that outputs a rolling-stock plan to a given week. A week was chosen out of the 52 weeks of the year to perform this study, in particular week 28 of the technical plan by Méchain (2017).

Tables 12 to 19 provide values for the parameters used in the mathematical model relative to the case study.

Table 12 - Information concerning stations

Station Name	Station Number, s	Minimal Turning Time, TM_s (min)
Roma-Areeiro	1	1
Entrecampos	2	1
Sete-Rios	3	1
Campolide	4	1
Pragal	5	1
Corroios	6	1
Foros de Amora	7	1
Fogueteiro	8	1
PMC (depot)	9	1
Coina	10	1
Penalva	11	1
Pinhal-Novo	12	1
Venda do Alcaide	13	1
Palmela	14	1
Setúbal	15	1

In Table 12, the first column gives the stations name, the next one their corresponding number and the last column their associated minimal turning time (in minutes). All stations, except PMC (depot) are the stations where the service tasks can start and end, or in other words, where there is an entrance and exit of passengers. PMC is the depot station, where only empty trains (without passengers) can enter to perform maintenance. As showed, the minimal turning time is equal for all stations, as 60 seconds are considered enough time for the entrance and exit of passengers and also for an eventual change of the train driver, apart from the station. As stated, this model aims to homogenise the turning times between two successive tasks, to avoid delays spreading. Thus, 1-minute turning times are unlikely to occur in the solution.

Table 13 - Pairs of stations between which there can exist dead-headings.

$W_{s,s'}$		s'														
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
	1	0	0	0	0	1	0	0	1	1	1	0	0	0	0	1
	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	5	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
s	**8**	1	0	0	0	0	0	0	0	0	1	0	0	0	0	1
	9	1	0	0	0	0	0	0	0	0	1	0	0	0	0	1
	10	1	0	0	0	0	0	0	1	1	0	0	0	0	0	1
	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	15	1	0	0	0	0	0	0	1	1	1	0	0	0	0	0

In Table 13, s and s' are respectively the departure and arrival stations of a possible dead-heading. If the value of $W_{s,s'}$ equals zero, a dead-heading between stations s and s' is not possible. Otherwise, its value would be equal to one. As showed, the use of dead-headings in the solution is quite limited. The reason is that Fertagus does not own the railway line and must pay infrastructure charges to the infrastructure manager, IP. This also means that Fertagus trains are not the only trains running over the line between Roma-Areeiro and Setúbal and so, dead-heading are to be avoided in order to minimize the traffic in the line. It is noteworthy that only Roma-Areiro, Coina and Setúbal are able to be linked to the depot.

Table 14 - Length of dead-headings.

$CW_{s,s'}$ (km)		s'														
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
	1	0	1.13	2.84	4.04	11.68	16.78	19.38	22.12	25.6	27.32	32.62	41.42	45.23	49.33	54.16
	2	1.13	0	1.71	2.91	10.55	15.65	18.25	20.99	0	26.19	31.49	40.29	44.1	48.2	53.03
	3	2.84	1.71	0	1.2	8.84	13.95	16.55	19.28	0	24.49	29.79	38.59	42.39	46.49	51.32
	4	4.04	2.91	1.2	0	7.65	12.75	15.35	18.09	0	23.29	28.59	37.39	41.19	45.29	50.12
	5	11.68	10.55	8.84	7.65	0	5.1	7.7	10.44	0	15.64	20.94	29.74	33.54	37.64	42.47
	6	16.78	15.65	13.95	12.75	5.1	0	2.6	5.34	0	10.54	15.84	24.64	28.44	32.54	37.37
	7	19.38	18.25	16.55	15.35	7.7	2.6	0	2.74	0	7.94	13.24	22.04	25.84	29.94	34.77
s	**8**	22.12	20.99	19.28	18.09	10.44	5.34	2.74	0	0	5.2	10.5	19.3	23.1	27.2	32.03
	9	25.6	0	0	0	0	0	0	0	0	1.7	0	0	0	0	28.6
	10	27.32	26.19	24.49	23.29	15.64	10.54	7.94	5.2	1.7	0	5.3	14.1	17.9	22	26.83
	11	32.62	31.49	29.79	28.59	20.94	15.84	13.24	10.5	0	5.3	0	8.8	12.6	16.7	21.53
	12	41.42	40.29	38.59	37.39	29.74	24.64	22.04	19.3	0	14.1	8.8	0	3.8	7.9	12.73
	13	45.23	44.1	42.39	41.19	33.54	28.44	25.84	23.1	0	17.9	12.6	3.8	0	4.1	8.93
	14	49.33	48.2	46.49	45.29	37.64	32.54	29.94	27.2	0	22	16.7	7.9	4.1	0	4.83
	15	54.16	53.03	51.32	50.12	42.47	37.37	34.77	32.03	28.6	26.83	21.53	12.73	8.93	4.83	0

Table **14** shows the distance in kilometres between stations s and s'. $CW_{s,s'}$ is set as zero if there is no information on that distance and if a dead-heading between stations s and s' is not possible. Nevertheless, the opposite is not necessarily true. The first and last station of the line are 54.16 kilometres away from each other and the distance between the two stations that link the north and south banks from the city (Campolide and Pragal) is around 8 kilometres

Table 15 - Duration of dead-headings.

$DW_{s,s'}$ (min)		s'														
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
s	**1**	0	0	0	0	16	0	0	22	24	26	0	0	0	0	45
	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	5	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	8	22	0	0	0	0	0	0	0	0	5	0	0	0	0	21
	9	24	0	0	0	0	0	0	0	0	2	0	0	0	0	21
	10	26	0	0	0	0	0	0	5	2	0	0	0	0	0	17
	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	15	45	0	0	0	0	0	0	21	21	17	0	0	0	0	0

Table 15 shows the duration of a dead-heading between stations s and s' (in minutes). $DW_{s,s'}$ is set to zero if there is no information on the duration and if case a dead-heading between stations s and s' is not possible. It takes 45 minutes to link the terminal stations of the line through a dead-heading.

Table 16 - Constants used.

Constant	Unit	Value
NU	---	17
NS	---	15
NT	---	790
ND	day	5
NM	---	14
LN	---	10000
PW	---	850
$PTHOM$	---	300
$PTZM$	---	850

In Table 16, all the constants are shown. By order, the number of train units considered to be available to perform the current operation schedules (and consequently of roster rows) are the 17 trains referred before (one train is used as backup train in case one of the others fails). Although the present work aims to minimize the probability of needing an extra unit to cover eventual disturbances in the service tasks fulfilment, it does not consider the probability of failure of a unit. So, the question of whether this could be done differently is left for further research. The number of stations is a sum of the 14 stations of the line plus the depot station (PMC), i.e. 15 stations. The number of real tasks equals 790 tasks, performed during the 5 working days of the week. Then, the number of days available for maintenance corresponds to the 5 working days of the week, which means that maintenance actions can be performed every day and the number of maintenance actions that can be performed in the depot are the 14 tasks referred before. LN is the large number to be used in one of the constraints regarding maintenance and is suitable for the size of the case study. Finally, the weights of the different terms of the objective function: the weight associated with dead-headings equals 850, the weight associated with turning times equals 300 and the weight associated with shuntings for maintenance purpose equals 850. PW and $PTZM$ are equal since they are equally relevant.

Figure 12 shows part of the actual rolling-stock plan used by Fertagus. In particular, it refers to four days of the week, namely: Monday, Tuesday, Wednesday and Thursday. Friday presents some differences. The service tasks are represented in Blue and Black lines. One line if only one train unit is needed and a double line if two unites are required for demand purposes, so DEM_i can take either the value 1 or 2. No maintenance actions are assigned to the units. Instead, for some of the units there is a period in the middle of the day, reserved for maintenance. The dead-headings needed for the present plan are represented in pink lines. Double lines if the dead-heading refers to two units. The service tasks to be covered and associated information were taken out of such rolling-stock plans provided by Fertagus.

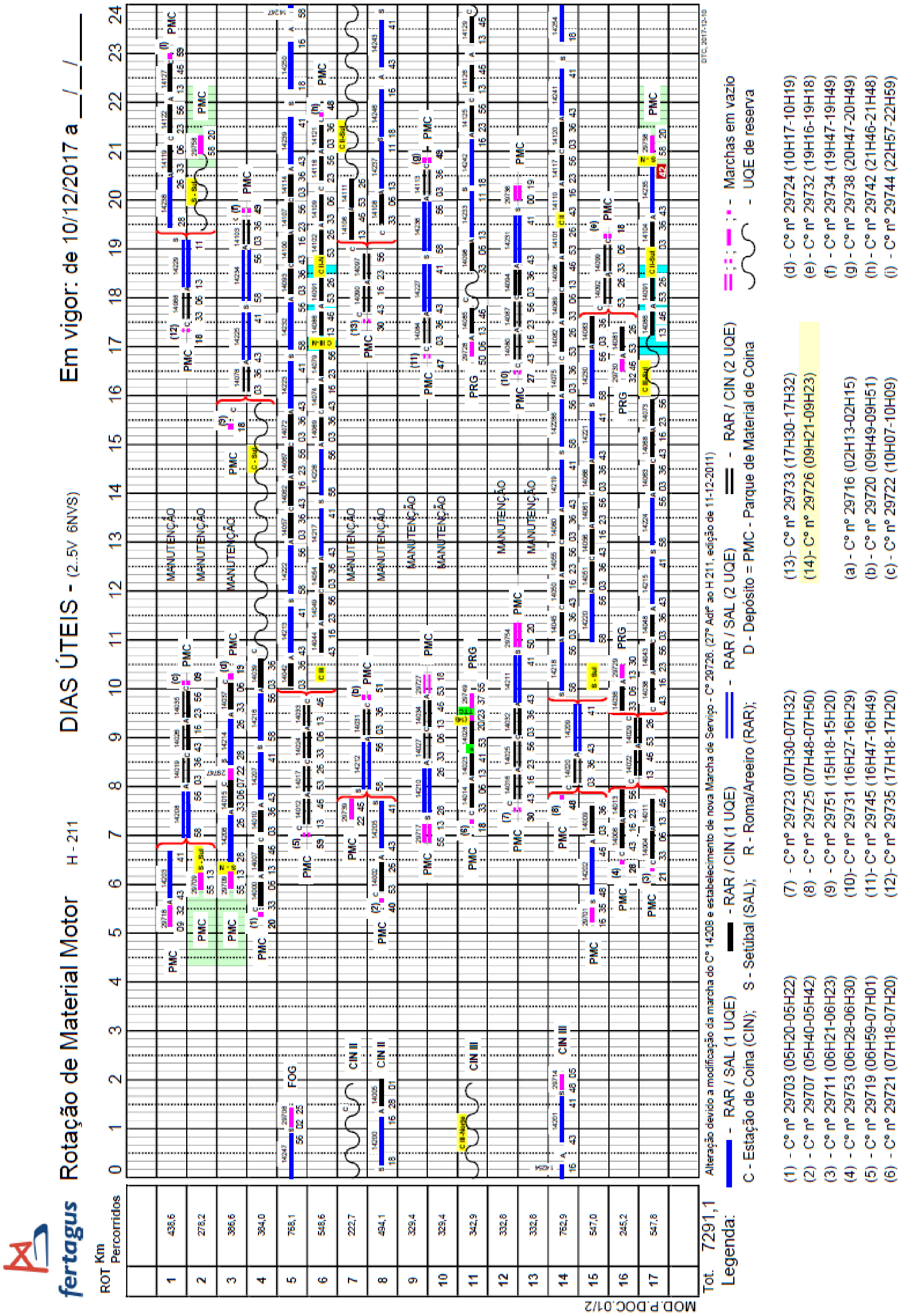

Figure 12 - Rolling-stock plan used by Fertagus relative to: Monday, Tuesday, Wednesday and Thursday.

Table 17, is as small excerpt of the table used to gather all information concerning all tasks of the study. The complete table is not presented, because of its extensive size. Just like Table 7 of the illustrative example, this table identifies the various tasks on the first column. The next columns give the required number of units, which is either 1 or 2, the maximal number of units, which is always 2, the departure station, the arrival station, the departure time and arrival time of a task. Tasks 1 to 790 are real tasks. The last 30 tasks are virtual tasks, and for that reason only have a departure and arrival station (the other values are zero). The departure and arrival times are in minutes.

Table 17 - Information about tasks (excerpt)

Task (T_i)	DEM_i	CAP_i	Sd_i	Sa_i	Dd_i (min)	Da_i (min)
1	1	2	1	15	343	401
2	2	2	15	1	418	476
3	2	2	1	10	483	516
4	2	2	10	1	523	556
5	2	2	1	10	563	596
6	2	2	10	1	1053	1086
7	2	2	1	15	1093	1151
8	1	2	15	1	1168	1226
9	1	2	1	10	1233	1266
10	1	2	10	1	1283	1316
11	1	2	1	10	1333	1366
12	1	2	15	15	1152	1258
13	1	2	15	1	388	446
14	1	2	1	10	453	486
15	1	2	15	1	508	566
16	1	2	1	10	573	606
17	2	2	10	1	963	996
18	2	2	1	15	1003	1061
19	2	2	15	1	1078	1133
20	2	2	1	10	1143	1176
			(...)			
817	0	0	12	12	0	0
818	0	0	13	13	0	0
819	0	0	14	14	0	0
820	0	0	15	15	0	0

Table 18 - Maintenance actions that need to be performed on week 28

$KM_{k,m}$		m													
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
k	**1**	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	2	1	0	0	0	0	0	0	0	0	0	0	1	1	0
	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	13	1	0	0	0	0	0	0	0	0	0	0	0	0	0
	14	1	0	0	0	0	0	0	0	0	0	0	0	0	1
	15	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0

In Table 8, k and m are respectively the train units and maintenance actions. $KM_{k,m}$ equals one when a maintenance action must be performed on a specific unit. The possible maintenance actions to be performed include among other activities, sets of inspections (motor block, pressure check, etc.) and check-ups (doors, lubrification, electric system, etc.). Specifically, unit 2 must perform three kinds of maintenance actions (1, 12 and 13), unit 13 must perform only maintenance actions 1 and unit 14 must perform two kinds of maintenance actions (1 and 14). The other units have no planned maintenance actions for the given week. Therefore, unit 2, unit 13 and unit 14 must go to the depot at least once, depending on other constraints, to satisfy the maintenance requirements.

Table 19 - Duration and working load of maintenance actions

Maintenance Action, m	MT_m **(min)**	AW_m **(min)**
1	150	744
2	420	1680
3	210	840
4	210	840
5	276	840
6	186	744
7	186	744
8	186	744
9	186	744
10	186	744
11	420	840
12	53	210
13	53	210
14	60	60

In Table 9, the first column identifies the different kinds of maintenance actions. The second column exhibits their duration and the third column their working load. Since the working load and the duration have a known relation ($duration = \frac{working\ load}{working\ persons}$), the number of workers needed for on each maintence task can be calculated. The maximum number of working persons available for preventive maintenance in Fertagus workshops is 5.

6 Results and Discussion

In chapter 6, the results for comparison with Fertagus' current plan are firstly presented and discussed. Then, several studies and analysis can be found. Several tests were also carried out to assess the impact of different weights used in the objective function. All model runs were carried out on the same computer, which is equipped with an installed memory of 16 GB (RAM), a 4.20 GHz processor and a 64-bit Operating System.

6.1 Robust Rolling-stock Plan

The present study can be divided into two problems: i) the construction of a robust rolling-stock plan and ii) the inclusion of preventive maintenance actions into that plan. Firstly, the robust rolling-stock plan is analysed separately.

6.1.1 Results Comparison with Fertagus' Plan

In order to compare the results of the model with the current plan used by Fertagus, it was decided to run the model without the maintenance actions first. Fertagus' plan does not include specific maintenance actions allocated to a given unit, but instead it has periods reserved for the maintenance of specific units (also known as maintenance slots). These periods were added to the model to perform this study, otherwise the comparison of results would not be fair, because the model could overlap service tasks with those periods, i.e. the model would be less restricted. For this implementation, the periods that a unit must spend in the depot for maintenance were given to the program, in a similar way that service tasks were defined (Table 20). The departure and arrival stations of these special tasks are the same (depot) and the demand is always 1 unit. Thus, only the departure and arrival times vary. This means that a unit stays at the depot and cannot cover any service task, while performing its maintenance slot. It restricts the construction of the roster.

Table 20 – Information concerning the periods a unit must spend in the depot for maintenance

Task (T_i)	DEM_i	CAP_i	Sd_i	Sa_i	Dd_i (min)	Da_i (min)
159	1	2	9	9	609	1038
160	1	2	9	9	609	1038
161	1	2	9	9	619	918
162	1	2	9	9	591	1052
163	1	2	9	9	591	1052
164	1	2	9	9	618	1007
165	1	2	9	9	618	1007
166	1	2	9	9	680	987
167	1	2	9	9	680	987

The program was run only for Monday to perform this comparison, as it was verified to be enough to understand if the roster varies or not. The results obtained for the rolling-stock plan were displayed by a data file (Figure 13), just like in the illustrative example. Due to the size of the plan, relative to the 17 train units, only the row of units 1 and 2 were chosen to be presented, to perform a proper analysis and comparison of results. As the Figure 13 shows, 2 dead-heading were introduced to fulfil all tasks of unit 2. For unit 1, no dead-headings were needed. It is also noteworthy that unit 2 went to the depot during the period reserved for its maintenance (task 165).

```
Train Unit #1
Pair of tasks [26-48] was performed
Pair of tasks [42-43] was performed
Pair of tasks [43-121] was performed
Pair of tasks [48-49] was performed
Pair of tasks [49-50] was performed
Pair of tasks [50-51] was performed
Pair of tasks [51-52] was performed
Pair of tasks [52-53] was performed
Pair of tasks [53-154] was performed
Pair of tasks [56-156] was performed
Pair of tasks [74-197] was performed
Pair of tasks [77-78] was performed
Pair of tasks [78-79] was performed
Pair of tasks [79-26] was performed
Pair of tasks [121-74] was performed
Pair of tasks [154-56] was performed
Pair of tasks [156-42] was performed
Pair of tasks [182-77] was performed

Train Unit #2
Pair of tasks [2-3] was performed
Pair of tasks [3-4] was performed
Pair of tasks [4-5] was performed
Pair of tasks [5-165] was performed
Pair of tasks [57-90] was performed
Pair of tasks [73-76] was performed
Pair of tasks [76-192] was performed
Pair of tasks [90-91] was performed
Pair of tasks [91-73] was performed
Pair of tasks [155-57] was performed
Pair of tasks [165-155] was performed
Pair of tasks [182-2] was performed
A dead heading was performed between tasks [5-165]
A dead heading was performed between tasks [165-155]
```

Figure 13 - Results of the rolling-stock plan, displayed by the data file

The row of unit 1, presented in Figure 14, was constructed with the information from Figure 13. Some information was added, to compare the results with Fertagus' current operational plan, displayed in Figure 12. In Figure 14, the first row of information gives the tasks that are covered by unit 1. In total 17 service tasks were covered, as task 182 and 197 are not accounted for, because they are virtual tasks. The second row of information shows the units that cover the same tasks as unit 1, in Fertagus' plan. As showed, the tasks now assigned to unit 1 are covered by six different units. Tasks 77, 78 and 79 must be covered by 2 units, due to their associated demand, and therefore are covered in the model by unit 1 coupled with unit 9. The third and fourth rows of information give the turning times (time between the arrival time of a task and the departure time of the next task) in minutes for each pair of real tasks covered, i.e. the time the unit is immobilized between tasks. Since tasks 182 and 192 are virtual tasks, the first and last pairs of tasks do not have an associated turning time.

Task:	182	77	78	79	26	48	49	50	51
Unit (Fertagus):	BVT	9+10			4	6			
Pairs of Tasks:		[182-77]	[77-78]	[78-79]	[79-26]	[26-48]	[48-49]	[49-50]	[50-51]
Turning Time (min):			7	7	17	7	7	7	7

52	53	154	56	156	42	43	21	74	197
6		17	6	17	5		14	8	EVT
[52-53]	[53-154]	[154-56]	[56-156]	[156-42]	[42-43]	[43-121]	[121-74]	[74-197]	
7	21	1	7	17	7	67	67		

Figure 14 - Row of unit 1, constructed with the information from **Figure 13,** and some information for comparison with Fertagus' operational plan displayed in **Figure 12**. BVT and EVT mean beginning and ending virtual tasks, respectively.

Although only one row is compared, it is enough to clarify that the results differ a lot from Fertagus' plan. One reason is that the model avoids the use of dead-heading for a more economical solution.

Table 21 shows information relative to the dead-headings performed by all the 17 units, both for the model and Fertagus' plan. The model uses almost the same number of dead-headings as Fertagus', 22 and 23, respectively. However, the total distance covered by all units is 64% less, which has a big impact on costs. The objective function used by the railway manager may not prioritize the avoidance of dead-headings, and furthermore, Fertagus' solution may also be restricted by the fact that the infrastructure is shared with other train operating companies, and thus some of the planned dead-headings might not be possible due to already existent railway traffic. Another reason is the concern of the model for a robust solution. As far as it does not deteriorate much the secondary costs, the model will seek for the turning times homogenization.

Table 21 – Information concerning all dead-headings performed, both for the model and Fertagus' operational plan for a single day.

Pairs of Stations	Dead-heading Distance (km)	Fertagus	Model
Roma-Areeiro - Pragal	11.68	4	4
Roma-Areeiro - PMC	25.6	2	0
Coina - PMC	1.7	14	16
Setúbal - PMC	28.6	2	2
Coina - Setúbal	26.83	1	0
	Total Dead-heading Distance (km):	**206**	**131**

6.1.2 Analysis of the Total Cost as Function of the Weight associated with Turning Times

A sensitivity analysis of the weight associated with turning times in the objective function ($PTHOM$) was carried out, to check if the values that were used by Tréfond et al. (2017) for the French company SNFC fit Fertagus' case study. In order to study the influence of this parameter on the total cost of the objective function, the value of the weight associated with dead-heading (PW) was fixed in 1500 and $PTHOM$ varied from 12 to 7500, as Table 22 shows. $PTZM$ is not referred in the table, because it is associated with the allocation of maintenance actions, and therefore is not used in the current section. From the relation between $PTHOM$'s variation in percentage and the total cost variation in percentage, it clear that the variation of total cost is damped with respect to the turning times variation. It means that the avoidance of the dead-headings is prioritized to the homogenization of turning times. This is the desired result, which guarantees that turning times do not deteriorate the secondary costs. They are to be minimized, but this minimization must be properly balanced. Thus, the value of 300 for $PTHOM$ fits the actual case study.

Table 22 - Relation between the variation of the weight associated with turning times ($\boldsymbol{PTHOM}$) and the variation of total cost of the objective function

$\boldsymbol{PTHOM}$	$\boldsymbol{PTHOM}$ **variation (%)**	**Total Cost**	**Total Cost variation (%)**
12	-96	196914	-3.9
36	-88	197596	-3.6
60	-80	198268	-3.3
150	-50	200808	-2.0
300	---	204961	---
600	+100	213243	+ 4.0
1500	+400	238086	+16.2
7500	+2400	403712	+97.0

6.1.3 Results for one week

A version of the model only relative to the construction of the robust rolling-stock plan was run for the 5 working days of the week, to analyse its costs, size and computational performance.

As Figure 15 shows, the minimum cost is 372 319. As expected, the term relative to secondary costs has by far the biggest impact, contributing to 94% of the total cost. The robustness indicator term represents 6% of the total cost.

It took about 52 minutes (3125.3 seconds) for the model run to be completed (Figure 16). The gap presented is considered as null, because its value is negligible (7.6 x 10^{-11} %), having no impact on the results. It means that the best bound for the objective function that was found corresponds to the best solution. As expected, the computational time increased, because the number of variables also increased. The matrix takes into account 5 273 315 variables, related with the size of the decision variables, which are reduced to 5 213 220 after the pre-solving stage. In fact, this computational time is quite acceptable, given the number of variables of the problem. It is also due to the data pre-processing, which compares all potential pairs of tasks and eliminates the pairs that do not present the requirements to be linked. It reduces considerably the size of the problem, making the model quite robust on a computational perspective.

```
The minimum cost is : 372319
Secondary Costs :350400
Robustness Indicator : 21918.9
Begin running model
End running model
```

Figure 15 - Minimum cost obtained

Matrix:		Presolved:	
Rows(constraints):	28661	Rows(constraints):	27684
Columns(variables):	5273315	Columns(variables):	5213220
Nonzero elements:	15745944	Nonzero elements:	10506289
Global entities:	5273315	Global entities:	5213220
Sets:	0	Sets:	0
Set members:	0	Set members:	0

Overall status: **Finished global search.**

LP relaxation:		Global search:	
Algorithm:	**Simplex primal**	Current node:	5129
Simplex iterations:	0	Depth:	1
Objective:	372319	Active nodes:	0
Status:	Unfinished	Best bound:	372319
Time:	161.5s	Best solution:	372319
		Gap:	7.62149e-011%
		Status:	Solution is optimal.
		Time:	3125.3s

Figure 16 – Model run information

6.2 Inclusion of the Preventive Maintenance Actions

In this section, the version of the model that is run also contemplates the preventive maintenance actions that should be performed. It corresponds to the complete version of the model and the actual proposal.

6.2.1 Results for one week

Several attempts were carried out to run the model for the time period of 5 working days (i.e., Monday, Tuesday, Wednesday, Thursday and Friday), however the size of the problem becomes too large and the computational capacity available is not enough to run the analysis. The calculation of the size of the problem is presented in Table 23, since it was not possible to obtain it by the program. A total of 159 159 015 variables have to be processed, about 30 times more than the number of variables of the problem without maintenance actions. In fact, the problem is the decision variable $yM_{k,i,j,m}$, which is responsible for 93,3% of the number of variables and so, it is responsible for this abrupt growth in the matrix size. The computer simply gets out of memory (Figure 17).

Table 23 – Calculus of matrix column for a 5 days plan

Set	Size	Decision Variables	Size	Size of the Columns
$k = \{1, \dots, 17\}$	17	$x_{k,i}$	$17 * 790 = 13\,430$	$159\,159\,015$
$i = \{1, \dots, 790\}$	790	$y_{k,i,j}$	$17 * 790 * 790 = 10\,609\,700$	
$j = \{1, \dots, 790\}$	790	$yM_{k,i,j,m}$	$17 * 790 * 790 * 14 = 148\,535\,800$	
$m = \{1, \dots, 14\}$	14	$zM_{k,d}$	$17 * 5 = 85$	
$d = \{1, \dots, 5\}$	5			

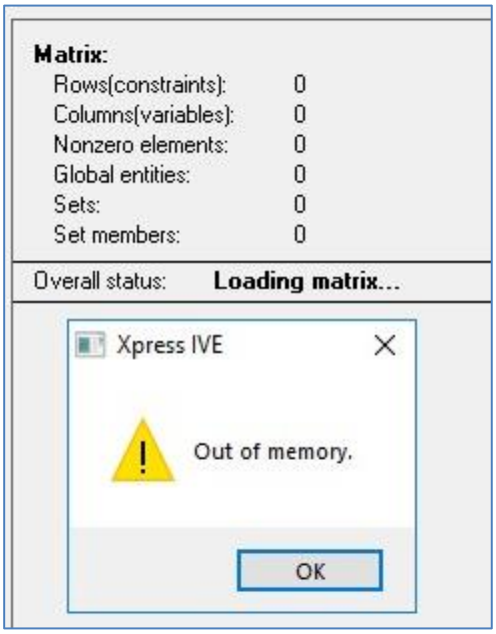

Figure 17 - Out of memory message from the stats window of Fico Xpress

Upon this limitation, a more compact implementation of the data pre-processing is suggested for further research and an even more compact version of the model is developed.

As an alternative, it was chosen to present the results for a 3-day plan, reducing in this way the size of the problem (but not its complexity). The rosters obtained for the units that must perform maintenance actions are presented in Figure 18, Figure 19 and Figure 20. Unit 2 performs all its maintenance actions on Tuesday (day 2) and both unit 13 and 14 perform their maintenance actions on Monday (day 1).

```
Train Unit #2
Pair of tasks [33-150] was performed
Pair of tasks [53-115] was performed
Pair of tasks [68-227] was performed
Pair of tasks [115-116] was performed
Pair of tasks [116-117] was performed
Pair of tasks [117-118] was performed
Pair of tasks [118-68] was performed
Pair of tasks [137-138] was performed
Pair of tasks [138-139] was performed
Pair of tasks [139-140] was performed
Pair of tasks [140-141] was performed
Pair of tasks [141-33] was performed
Pair of tasks [150-53] was performed
Pair of tasks [190-266] was performed
Pair of tasks [220-221] was performed
Pair of tasks [221-190] was performed
Pair of tasks [227-228] was performed
Pair of tasks [228-220] was performed
Pair of tasks [250-280] was performed
Pair of tasks [259-260] was performed
Pair of tasks [260-261] was performed
Pair of tasks [261-262] was performed
Pair of tasks [262-250] was performed
Pair of tasks [266-259] was performed
Pair of tasks [280-318] was performed
Pair of tasks [318-319] was performed
Pair of tasks [319-320] was performed
Pair of tasks [320-321] was performed
Pair of tasks [321-425] was performed
Pair of tasks [322-323] was performed
Pair of tasks [323-324] was performed
Pair of tasks [324-361] was performed
Pair of tasks [361-504] was performed
Pair of tasks [425-446] was performed
Pair of tasks [446-470] was performed
Pair of tasks [470-322] was performed
Pair of tasks [484-137] was performed
A maintenance slot of kind [1] was covered between tasks [266-259]
A maintenance slot of kind [12] was covered between tasks [266-259]
A maintenance slot of kind [13] was covered between tasks [266-259]
This unit went to the depot for maintenance on day [2]
```

Figure 18 - Resuls of the roster of unit 2, displayed by the data file

```
Train Unit #13
Pair of tasks [2-3] was performed
Pair of tasks [3-4] was performed
Pair of tasks [4-5] was performed
Pair of tasks [5-111] was performed
Pair of tasks [10-95] was performed
Pair of tasks [42-59] was performed
Pair of tasks [59-10] was performed
Pair of tasks [89-42] was performed
Pair of tasks [95-179] was performed
Pair of tasks [111-132] was performed
Pair of tasks [132-133] was performed
Pair of tasks [133-134] was performed
Pair of tasks [134-89] was performed
Pair of tasks [173-174] was performed
Pair of tasks [174-194] was performed
Pair of tasks [179-228] was performed
Pair of tasks [194-311] was performed
Pair of tasks [205-318] was performed
Pair of tasks [228-173] was performed
Pair of tasks [238-239] was performed
Pair of tasks [239-240] was performed
Pair of tasks [240-241] was performed
Pair of tasks [241-252] was performed
Pair of tasks [252-205] was performed
Pair of tasks [311-238] was performed
Pair of tasks [318-319] was performed
Pair of tasks [319-320] was performed
Pair of tasks [320-321] was performed
Pair of tasks [321-464] was performed
Pair of tasks [322-323] was performed
Pair of tasks [323-328] was performed
Pair of tasks [328-439] was performed
Pair of tasks [367-429] was performed
Pair of tasks [371-322] was performed
Pair of tasks [392-499] was performed
Pair of tasks [429-371] was performed
Pair of tasks [439-392] was performed
Pair of tasks [464-367] was performed
Pair of tasks [489-2] was performed
A maintenance slot of kind [1] was covered between tasks [5-111]
This unit went to the depot for maintenance on day [1]
```

Figure 19 - Resuls of the roster of unit 13, displayed by the data file

```
Train Unit #14
Pair of tasks [1-62] was performed
Pair of tasks [17-143] was performed
Pair of tasks [32-108] was performed
Pair of tasks [62-63] was performed
Pair of tasks [63-32] was performed
Pair of tasks [64-65] was performed
Pair of tasks [65-66] was performed
Pair of tasks [66-83] was performed
Pair of tasks [83-94] was performed
Pair of tasks [94-124] was performed
Pair of tasks [108-17] was performed
Pair of tasks [124-160] was performed
Pair of tasks [143-64] was performed
Pair of tasks [160-161] was performed
Pair of tasks [161-162] was performed
Pair of tasks [162-163] was performed
Pair of tasks [163-287] was performed
Pair of tasks [175-301] was performed
Pair of tasks [195-175] was performed
Pair of tasks [205-393] was performed
Pair of tasks [222-223] was performed
Pair of tasks [223-315] was performed
Pair of tasks [225-252] was performed
Pair of tasks [252-205] was performed
Pair of tasks [287-195] was performed
Pair of tasks [301-222] was performed
Pair of tasks [315-225] was performed
Pair of tasks [342-468] was performed
Pair of tasks [355-356] was performed
Pair of tasks [356-357] was performed
Pair of tasks [357-374] was performed
Pair of tasks [362-490] was performed
Pair of tasks [374-474] was performed
Pair of tasks [393-394] was performed
Pair of tasks [394-395] was performed
Pair of tasks [395-342] was performed
Pair of tasks [468-355] was performed
Pair of tasks [474-362] was performed
Pair of tasks [475-1] was performed
A maintenance slot of kind [1] was covered between tasks [108-17]
A maintenance slot of kind [14] was covered between tasks [108-17]
This unit went to the depot for maintenance on day [1]
```

Figure 20 - Resuls of the roster of unit 14, displayed by the data file

It took about 20 minutes for the model run to be completed, with a total of 62 363 412 variables (Figure 21). Once again, the gap presented is considered as null, because its value is negligible and therefore the solution presented is the optimal solution. The fact that the solutions are always optimal allows to conclude that this model is solid and that apart from the memory run out problem it is quite fast on finding solutions (considering the size of the problem). It is frequent that optimization models take too long to present results and for that reason have to be interrupted before an optimal solution is found. Therefore, analysis of the optimality gap as a function of computational time is usually carried. For this model, it was not considered necessary to present such analysis, since the longest model run took less than one hour to be completed.

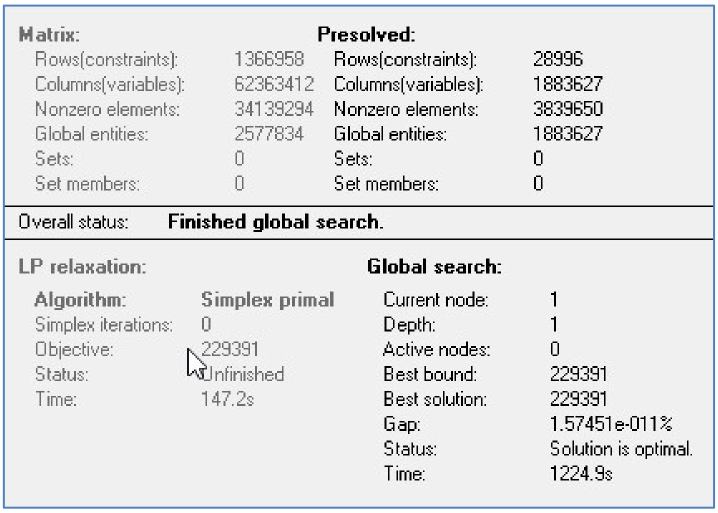

Figure 21 - Model run information

As Figure 22 shows, the best solution is a roster with a cost of 223 991. As usual, the term relative to secondary costs has by far the biggest impact, contributing to 93.7% of the total cost. The robustness indicator term represents 6% of the total cost and the shuntings for maintenance 0.3%.

```
The minimum cost is : 223991
Secondary Costs : 210240
Robustness Indicator : 13151.3
Shuntings for Maintenance : 600
Begin running model
End running model
```

Figure 22 - Minimum cost obtained

6.2.2 Analysis of the weights associated with Secondary costs and Shuntings for Maintenance

Due to the fact that the last term of the objective function related to the shuntings for maintenance is an actual contribution of this research, a sensitivity analysis was performed relative to the relation between the weights of the secondary costs (PW) and the shuntings for maintenance ($PTZM$). The methodology followed consisted of keeping the sum of the values of all weights fixed, as well as the weight of the robustness indicator ($PTHOM$), and changing only the values of PW and $PTZM$ (Table 24). Four sets of values were tested (scenarios I, II, III and IV).

Table 24 – Sets of tested values for the analysis of $\boldsymbol{PW}$ and $\boldsymbol{PTZM}$

	I	II	III	IV
$\boldsymbol{PW}$	1500	850	200	30
$\boldsymbol{PTHOM}$	300	300	300	300
$\boldsymbol{PTZM}$	200	850	1500	1670
Sum	**2000**	**2000**	**2000**	**2000**

The results from the tests are presented in Table 25 and refer to values of each cost component of the objective function.

Table 25 – Values of the different cost components of the objective function along the four tests

	I	II	III	IV
Secondary Costs	210240	119136	28032	4205
Robustness Indicator	13151	13151	13151	13151
Shuntings for Maintenance	600	2550	4500	5010
Total Cost	223991	134837	45683	22366

For an easier comparison of results, a graph is presented in Figure 23. First, the value of the robustness indicator cost remained unchanged along the tests, which indicates that the solutions found are the same. Moreover, the growth of the cost relative to shuntings for maintenance increased proportionally to the variation of its weight, just like the value relative to the secondary costs that decreased along the tests. These relations are linear. Furthermore, the value of total cost follows the reduction of the secondary costs. The results are in line to what was expected, i.e. the secondary costs are not deteriorated with the introduction of robustness and the secondary costs and shuntings for maintenance are lined up to the same solution. In fact, a unit that goes less times to the depot is a unit that covers less dead-heading to the depot. Altogether, it means that the solution of the model was not affected by the change of the weights, which indicates that the optimal solution is robust to different preferences of the decision maker between the three components of the objective function.

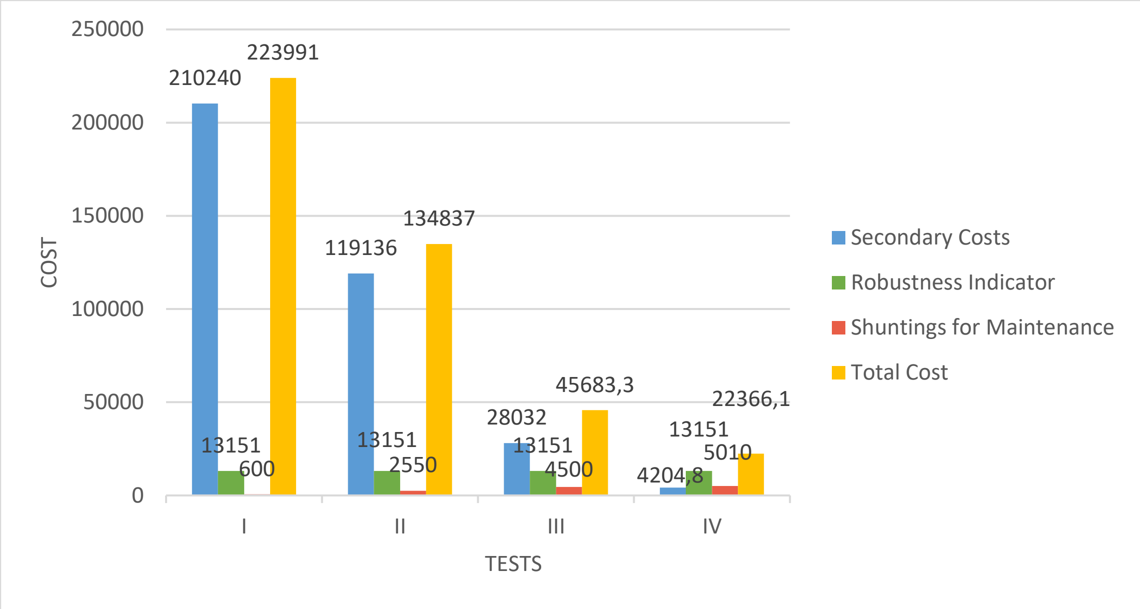

Figure 23 – Graph concerning the variation of costs components along the tested solutions.

7 Conclusion

In this final chapter, the main conclusions of the research conducted within the present dissertation can be found. Moreover, some limitations are identified and discussed, and possible steps for future research are also presented.

7.1 Contributions

The goal of the present dissertation was to create a mathematical model that would provide an optimal robust rolling-stock plan, capable of including the maintenance tasks from a technical plan relative to the different weeks of the year, in line with what has been proposed in Tréfond et al. (2017) and suggested as future research (namely the inclusion of the maintenance actions). Moreover, at the same time reduce the overall costs of the operation to a minimum. The mathematical model developed is successfully adapted to the specific case of Fertagus railway company and it is flexible enough to be modified and fit to the conditions of a different train operating company. Naturally, the adaptation of the model to Fertagus context required the collection of data related to its operational services and also relative to their maintenance operations in the depot. The extensive collection of this data provides a more complete case study, when compared for instance with the first approach followed in Méchain (2017). In fact, the first approach proposed was a technical maintenance plan with a planning horizon of a year and a time step of a week. In this model, the proposed model is an operational model with a planning horizon of a week and a time step of a minute.

The results showed that the optimal solution for rolling stock plan, without maintenance actions, provides a reduction of total deadheading distance of around 64%, which represents an important research opportunity to test whether or not current train schedules of Fertagus train operating company are suboptimal according to the preferences of the decision maker. These preferences, and even the components of the objective function might be different from the model presented adapted from Tréfond et al. (2017). Nevertheless, it is important to mention that such a reduction in the total deadheading distance may collide with slots made available to other train operators by the infrastructure manager. The assessment of such impacts is outside the scope of the present research.

The inclusion of the maintenance tasks in the proposed model led to a substantial increase in the number of decision variables, and in fact, it was showed that such increase makes a 5-day schedule impossible to run for the computer used, as it runs out of memory. Only a 3-day schedule was possible to run, and results were presented for that case. Moreover, sensitivity analysis on the weights of the different components of the objective function showed that the optimal solution found is not sensitive to significant variations of the weights.

As one of the goals of this research was to verify that the program could give an optimal feasible solution to the rolling-stock planning problem. This was shown to be possible through a successive and iterative process in the construction of the model. In fact, many unfeasible solutions were found, whenever the program was too restrictive. Therefore, the mathematical model is considered to answer its purpose and solve to optimality several real instances.

7.2 Limitations

This mathematical model enables to find an optimized rolling stock roster, but the size of the problem is limited by the computational capacity. A way to work around this problem could be to add even more restrictions in the phase of data pre-processing and implement them through successive steps. They would work as filters, progressively reducing the size of the problem before the phase of minimization of the objective function and computations of its constraints. This method requires that possible solutions of the problem (but of no interest) are excluded before the start of the actual optimization process and associated linear programming relaxation. A good starting point towards this idea would be reducing the possible pairs of tasks, by stating that tasks that are too far in time would not be linked in the optimal solution.

Moreover, having in mind that Fertagus has a small fleet (with only 18 train units), the adaptation of the present model to larger fleets would require the use of metaheuristics to be able to reduce the computational time. Finally, the interface relative to the model could be more user friendly and accordingly an automatically generated chart could be implemented for the presentation of the obtained rosters. The analysis of the roster would be much faster and facilitated.

Although the solution of the model was not affected by the change of the weights of the different components of the objective function, it does not mean that this behaviour is a property of the model. In fact, it may not happen in different scenarios, dissimilar to Fertagus case study. In other words, the optimal solution found for Fertagus case study is robust to different preferences of the decision maker between the three components of the objective function, but for different case studies the solutions may differ with the variation of the impact of each component of the objective function.

7.3 Future Research

A considerable improvement to this research would be the proposal of a method to promote a balanced usage of Fertagus fleet and consequently a balanced wear. The usage of the units could be measured by the travelled kilometres and a new term introduced in the objective function, concerning this number. An input parameter could also be added relative to the mileage account of a unit before the starting of a week. However, the travelled kilometres are not the best indicator of the usage of a unit. A good method to guarantee an equal wear is to assign similar services to all units. With the cyclic rooster proposed by Tréfond et al (2017), after a specific number of cycles, all units were subjected to the same services. However, it does not fit the actual case study, due to a non-repeatability of the maintenance tasks scheduled along the time. It would be wise to align the repeatability of maintenance tasks with the repeatability of service tasks along a time period of interest for the company (i.e. after that time period, all units would have suffered the same wear).

Regarding maintenance planning, it should be pointed that the costs of preventive maintenance only represent about a half of the costs of corrective maintenance. Corrective maintenance can only be predicted through a predictive model, and so corrective maintenance actions cannot be scheduled for an entire year, like preventive maintenance ones. Nevertheless, it would make sense to include corrective maintenance slots in the rolling-stock planning problem, at least for the time period of a week, and inform the decision maker on the robustness of such a plan with the corrective maintenance slots.

Comprehensive crew scheduling that takes into account the different skill of maintenance technicians and their experience is still missing in the current version of the model, and further research should include it. Finally, an intermediate model is missing that links the 1-day operational planning model proposed in this dissertation and the annual tactical plan proposed in Méchain (2017). Such intermediate model might allocate the weekly maintenance tasks into the different days using some criterion, minimizing or maximizing a certain objective function.

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